1. (a) Determine all complex values of \( z \) for which \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!} z^n \) converges.

(b) Find a power series \( \sum_{n=0}^{\infty} a_n z^n \) that converges for all complex \( z \) with \( |z| < 1 \) and diverges for all \( |z| \geq 1 \).

(c) Find a power series \( \sum_{n=0}^{\infty} b_n z^n \) that converges for at least one complex number \( z \) with \( |z| = 1 \) and diverges for at least one \( z \) with \( |z| = 1 \).

2. Determine the radius of convergence of the power series \( \sum_{n=1}^{\infty} a_n z^n \) for each of the following.

(a) \( a_n = (1 + 1/n)^{-n^2} \),

(b) \( a_{2m} = m^2, a_{2m+1} = 4^{-m} \),

(c) \( a_{2m} = (\log m)^m, a_{2m+1} \) arbitrary.

3. For \( z \in \mathbb{C} \), let \( e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \). Prove that \( e^\alpha e^\beta = e^{\alpha + \beta} \) for all \( \alpha, \beta \in \mathbb{C} \).

Hint: Theorem 3.50.

4. Suppose that \( \sum_{n=0} a_n z^n \) and \( \sum_{n=0} b_n z^n \) have radii of convergence \( R_1 \) and \( R_2 \), respectively. Prove that the radius of convergence of \( \sum_{n=0} a_n b_n z^n \) is at least \( R_1 R_2 \).

5. If \( \sum_{n} a_n z^n \) has radius of convergence \( R \), what are the radii of convergence of \( \sum_{n} a_n z^{2n} \) and \( \sum_{n} a_n^2 z^n \)?

6. For each of the following series, determine which values of \( z \) give convergence and which give divergence.

(a) \( \sum_{n=0} (\frac{z}{1+z})^n \),

(b) \( \sum_{n=0}^{\infty} \frac{z^n}{1+z^n} \).

(The case \( |z| = 1 \) is a bonus question for those familiar with the fact that in this case \( z = e^{i\theta} = \cos \theta + i \sin \theta \) with \( \theta \in \mathbb{R} \).)

Practice problems from Rudin (not to be handed in):

Chapter 3: #9, 10, 13.