Math 320 Assignment 1: Due Friday, September 18 at start of class

1. Rudin Chapter 1, #2.

2. Let \( E = \{ r \in \mathbb{Q} : r^3 < 27 \} \). Find \( \inf E \) and \( \sup E \) if they exist, and determine whether they are elements of \( E \). (Prove all your claims.)

3. Let \((S, <)\) be an ordered set and \( A \subset S \). Suppose that \( A \) contains a largest element, which we denote by \( \max A \).
   (a) Prove that \( A \) has exactly one largest element, i.e., that \( \max A \) is unique.
   (b) Prove that \( \sup A \) exists in \( S \) and equals \( \max A \).

4. Read the section on Fields, pp.5–8.
   Rudin Chapter 1, #3.

5. The real numbers \( \mathbb{R} \) are constructed in Theorem 1.19 as an ordered field which has the least-upper-bound property.
   Rudin Chapter 1, #5.

6. Let \( S_1 \) and \( S_2 \) be nonempty subsets of \( \mathbb{R} \) that are bounded above. Let \( S_1 + S_2 = \{ x+y : x \in S_1, y \in S_2 \} \) and \( S_1 - S_2 = \{ x - y : x \in S_1, y \in S_2 \} \). For each of the following statements, give a proof if it is true or a counterexample if it is false.
   (a) \( \sup(S_1 + S_2) = \sup S_1 + \sup S_2 \).
   (b) If \( S_2 \) is also bounded below then \( \sup(S_1 - S_2) = \sup S_1 - \sup S_2 \).