MIDTERM #1, MATH 317

Monday, February 6, 2006

Student No: ____________________ Name (Print): ____

1. (9 marks) Answer true or false to the following questions by putting either true or false in the boxes. If the answer is true give a proof or valid reason, and if the answer is false state why.

   (a) If C is a smooth space curve then $\mathbf{B}, \mathbf{N}, \mathbf{T}$, in that order, is a right hand system of mutually perpendicular unit vectors.

   (b) The space curve $C : x = \cos t, y = \sin^2 t, z = \sin t, -\infty < t < \infty$, is the intersection of the surfaces $y = z^2$ and $x^2 + z^2 = 1$.

   (c) If $\mathbf{r}(t)$ is a space curve such that $\mathbf{r}''(t)$ exists then $\dfrac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)] = \mathbf{r}(t) \times \mathbf{r}''(t)$.

2. (9 marks) The following questions require little or no computation. Enter your answers in the boxes, if provided.

(a) If the space curve $\mathbf{r}(t)$ has the property that $\mathbf{r}(t)$ is always perpendicular to $\mathbf{r}'(t)$ show that $|\mathbf{r}(t)|$ is a constant for all t.

(b) Evaluate the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$, where $\mathbf{F} =< xy, yz, zx >$ and C is the twisted cubic $\mathbf{r}(t) =< t, t^2, t^3 >,\ 0 \leq t \leq 1$.

$\boxed{}$

(c) Find the arc length of the curve $\mathbf{r}(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t \rangle,\ 0 \leq t \leq \pi$.

$\boxed{}$

3. (3 marks) Find the velocity, speed and acceleration of a particle whose position at time t is given by $\mathbf{r}(t) =< \cos t, t, \sin t >$.

---

4. (5 marks) Find $\mathbf{T}(t), \mathbf{N}(t), \mathbf{B}(t)$, at $t = 0$, for the space curve $\mathbf{r}(t) = \langle e^t, e^t \sin t, e^t \cos t \rangle$.

5. (4 marks) Determine if the vector field $\mathbf{F}(x, y) = (ye^x + \sin y)\mathbf{i} + (e^x + x \cos y + y)\mathbf{j}$ is conservative, and if so find **all** potential functions.

---

6. (4 marks) Prove that the vector field $\mathbf{F} = \dfrac{-y}{x^2 + y^2}\mathbf{i} + \dfrac{x}{x^2 + y^2}\mathbf{j}$, defined on the plane minus the origin, is not conservative.