(1) Probability distribution: Consider the two probability density functions $p_1(x) = c(1-|x|)$ and $p_2(x) = c(1-x^2)$ on the interval $[-1, 1]$. (See Figure 1 for a sketch.) (a) Find the value for the constant c in each case so that the probability distribution is **normalized** (i.e. the total probability is 1). (b) Find the average value (mean) for both of these functions. (Note: a careful use of symmetry will simplify your task.) (c) Find the median for each distribution.

(2) The probability that the height of a basketball player is x meters is given by the probability density distribution $p(x) = Cxe^{-x/2}$ where $0 < x < 3$.

(a) What is the constant C ?

(b) What is the most probable height ?

(c) What is the mean height?

(3) Suppose that the probability of jumping a distance x is $p(x) = \frac{\pi}{6}\sin(\frac{\pi}{3}x)$ for $0 \le x \le 3$.

(a) Explain the constant $\pi/6$ in front of the sine function.

(b) Compute and sketch the cumulative distribution function, $F(x)$ corresponding to this probability density.

(c) Find the zeroth, the first, and the second **moments** of this probability distribution, i.e. calculate $I_0 = \int_0^3 p(x)\,dx, I_1 = \int_0^3 x\,p(x)\,dx, I_2 = \int_0^3 x^2\,p(x)\,dx$.

(d) Find the mean, the variance, and the standard deviation of the jump distance.

(4) Consider the uniform probability distribution $p(x) = C, \quad a < x < b$. (a) Find the value of the constant C. (b) Compute the mean and the median of the distribution. (c) Find the variance and the standard deviation. (d) Sketch the cumulative distribution function, $F(x)$ for this probability density.

(5) Given the probability distribution $p(x) = C\frac{1}{(1+x)^2}$, defined on $0 < x < \infty$, (a) find the probability that x takes on values in the interval $[1, 4]$. (You will first have to find the value of the constant, C). (b) Sketch both $p(x)$ and $F(x)$, the cumulative distribution function, $F(x)$ for this probability density.

(6) According to L. Glass and M. Mackey, the authors of a recent book **From clocks to chaos; the rhythms of life** p 44, "The probability that a cell drawn at random from a large population has divided at time $t \ge 20$ (minutes) after birth is $p(t) = Re^{-R(t-20)} \quad t \ge 20$. (They assume that cells never divide before time $t = 20$ min.) (a) Find the mean time for division. (b) What fraction of the cells have not divided by time $t = 40$ min?

(7) The probability that seeds will be dispersed a distance x (meters) away from a parent tree is found to be $p(x) = Ce^{-x/10}$. (a) What is the mean dispersal distance? (b) What is the variance in the dispersal distance? (c) Seeds that fall right under the tree (for $x < 1$ meter) fail to thrive because of competition with the parent tree and shading that stunts the growth of the seedling. What fraction of the seeds will fail to thrive ? (Assume a one-dimensional arrangement.)

(8) Mortality is often highly age dependent. Suppose that the probability of dying at age x (in years) is $p(x) = C(1 + (x - 20)^2)$ where $0 < x < 100$. (a) Find the constant C. (b) Find the fraction of the population that has died by age 20. (c) Find the fraction of the population that has died by age 80. (d) What is the mean lifetime in this population?

(9) Quantum mechanics and electron clouds: Quantum mechanics tells us that we can never know with complete certainty where a particle is at any particular time. Instead, it gives us a probability density which describes the chances that a particle is at some position. Einstein, as well as many others, found this difficult to accept, hence his famous quote "God does not play dice with the universe." Nevertheless, quantum mechanics has proven to be a theory which is remarkably consistent with physical observations.

For instance, if we are interested in how far the electron in a hydrogen atom is away from its nucleus, the probability density function is $p(r) = 4r^2e^{-2r}$ where r is the distance measured in "Bohr radii". (One Bohr radius is the distance 5.29×10^{-11} meters.)

(a) Sketch the probability density function by determining $p(0), p'(r)$ and $\lim_{r\to\infty} p(r)$.
(b) Near which value of r is the electron most likely to be found?
(c) Find the mean (average value of the) distance.
(d) Find the cumulative distribution $D(r)$. What is the probability that the electron is found within 1 Bohr radius of the nucleus.

(10) The probability density of a light bulb failing at time t is $Ce^{-t/100}$ for t in days. (a) What is C? (b) What is the probability that a light bulb lasts at least 100 days? (c) What is the mean life of a light bulb? (d) The median? (e) The probability of lasting at least T days?