Homework Assignment #8

due in class on Friday, Mar. 28

Note: All homework assignments are due in class one week after being assigned. They must be on standard $8\frac{1}{2} \times 11$ paper and they must be stapled. Assignments which are not stapled will not be accepted. I will not bring a stapler to class. Please put your answers in the boxes and show your work in the spaces provided.

Name (Print): _____ Student No: _____

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1. [3 marks] A cylindrical water reservoir develops a leak so that the height $y(t)$ of water in the reservoir at time t satisfies the differential equation $\dfrac{dy}{dt} = -k\sqrt{y}$, where k is a positive constant.

(a) If $y(0) = y_0 > 0$, find the height of water in the tank as a function of time t.

(b) How long will it take for all the water to drain out?

2. [6 marks] Find the solution(s) of the following differential equations.

(a) $y''(x) + 5y'(x) + 6y(x) = 0$.

(b) $y''(x) + 5y'(x) + 6y(x) = 0$, $y(0) = 1$, $y'(0) = -1$.

(c) $y''(x) + 4y'(x) + 4y(x) = 0$.

(d) $y''(x) + 4y'(x) + 4y(x) = 0$, $y(0) = 1$, $y'(0) = 0$.

(e) $y''(x) + 2y'(x) + 2y(x) = 0$.

(f) $y''(x) + 2y'(x) + 2y(x) = 0$, $y(0) = 2$, $y'(0) = 1$.

3. [6 marks] Find the solution(s) of the following differential equations.

(a) $y''(x) + 2y'(x) - 8y(x) = x$.

(b) $y''(x) + 2y'(x) + y(x) = \sin x$, $y(0) = 1$, $y'(0) = 1$.

(c) $y''(x) - \lambda^2 y(x) = e^x$, $y(0) = 0$, $y'(0) = 0$, where $\lambda \neq 0, \pm 1$.

4. [5 marks] When a mass of 1 kg is attached to a spring it stretches a distance of 10 cm. Use $g = 9.8\ m/sec^2$ for the gravitational constant.

(a) Determine the spring constant k.

(b) Now a mass of 2 kg is attached to the spring and it is allowed to come to rest at equilibrium. Then it is compressed 10 cm and released with no velocity. Determine the displacement $y(t)$ at all times t assuming there is no friction.

(c) Now suppose the spring mass system in (b) is acted upon by an external force equal to $\sin t$ *newtons*. Determine the displacement $y(t)$ at all times t assuming the same initial conditions and no friction.