

Homework Assignment #6

due in class on Friday, March 14

Note: All homework assignments are due in class one week after being assigned. They must be on standard $8\frac{1}{2} \times 11$ paper and they must be stapled. Assignments which are not stapled will not be accepted. I will not bring a stapler to class. Please put your answers in the boxes and show all work in the spaces provided.

Name (Print): _____

Student No: _____

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1. [2 marks] Consider a breed of rabbits whose population $P(t)$ at time t satisfies the initial value problem $\frac{dP}{dt} = kP^2$, $P(0) = P_0$, where k and P_0 are positive constants. Show that the solution is $P(t) = \frac{P_0}{1 - kP_0t}$.

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2. [2 marks] A tank has the shape of the paraboloid obtained by rotating the parabola $y = x^2$, $0 \leq x \leq 8ft$, about the y -axis. How much work is done in filling the tank with oil ($\rho = 50 lb/ft^3$) if it is pumped from $1000ft$ below ground level?

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3. [2 marks] To compress a spring, whose natural length is $3ft$, to 75% of its length, a force of $20lb$ s is required. Determine the amount of work required to stretch this spring to $6ft$.

4. [8 marks] Find the solution(s) of the following differential equations.

(a) $\frac{dx}{dt} = 2x + 3$.

(b) $\frac{dv}{dt} = 2v - v^2$.

(c) $\frac{dx}{d\theta} = x \sin \theta \cos \theta$, $x(\pi/2) = -1$.

(d) $\frac{dy}{dx} = y \ln x$, $y(e) = -e$.

5. [3 marks] A chain of length L meters and mass M kg is hanging over the edge of a building under construction. Determine the amount of work required to pull the chain up to roof level. Assume the constant of gravity is $g = 9.8 \text{ m sec}^2$.

6. [3 marks] A conical tank, with radius r ft and height h ft, is resting on its base at ground level. Compute the amount of work done in filling this tank with water ($\rho = 62.4 \text{ lb/ft}^3$) assuming it is pumped from a reservoir 20 ft below ground level.