

HOMWORK ASSIGNMENT #5

due in class on Friday, October 18

Student No: _____ Name (Print): _____

Note: All homework assignments are due in class one week after being assigned. They must be on standard $8\frac{1}{2} \times 11$ size paper and they must be stapled. Assignments which are not stapled will not be accepted. I will not bring a stapler to class. Please enter your student number and name (as it appears on the registrar's list) in the spaces above. SURNAME FIRST IN CAPITALS, and given name second. Please put your answers in the boxes (if provided) and submit these pages for your assignment.

1. Each of the following questions can be done with little computation. Enter your answers in the boxes and show any work in the spaces provided.

Find derivatives of the following functions and simplify as much as possible:

(a) $f(x) = \sqrt{x} \tan(\sin x)$

(b) $f(x) = e^{\sqrt{x}} + e^{-\sqrt{x}}$.

(c) $f(x) = \ln(\ln x)$.

(d) $y = x^{\sin x}$.

(e) $y = \left(1 + \frac{1}{x}\right)^x$.

2. Show that $\frac{d}{dx} \ln(x + \sqrt{1 + x^2}) = \frac{1}{\sqrt{1 + x^2}}$.

3. Find the equations of the tangent lines to the graphs of $y = f(x)$ at $x = a$. Enter your answers in the boxes and show any work in the spaces provided.

(a) $f(x) = e^{x^2 - x}$, $a = 1$

(b) $f(x) = \frac{\ln x}{x^2}$, $a = e$.

(c) $f(x) = \frac{e^{x^2}}{\cos \pi x}$, $a = 1$.

4. Find all values of x where the graph of $y = x - \sin 2x$ has a horizontal tangent.

5. Carbon extracted from an ancient skull recently unearthed contained only $1/6$ as much ^{14}C as carbon extracted from present-day bone. How old is the skull?

6. Upon the birth of their first child a couple deposited \$5000 in a savings account that pays 6% annual interest compounded continuously. How much will the account contain when the child is ready to go to college at age 18?

7. The population of a certain town was 50,000 in 1980 and 70,000 in 2000. Assuming population growth is exponential determine what it will be in 2010.

8. The half life of radioactive cobalt is 5.27 years. If a certain region has 100 times the safe level of radioactive cobalt, how long will it take for the region to again be safe?

9. The population of a certain bacteria is known to increase 10-fold over a 24 hour period. Determine the doubling time.

10. Suppose a mutation of the bacterium *E. coli* produces a cell line that divides into 3 daughter cells every 30 minutes. How many cells would there be after 1 day?

11. C^{14} , a radioactive isotope of carbon, has a half life of 5730 years. (Note: this isotope is used in radiocarbon dating, a process by which the age of materials containing carbon can be estimated. W. Libby received the Nobel prize in chemistry in 1960 for developing this technique.)

(a) Determine how long it takes for a sample to fall to 0.001 of its original level of radioactivity.

(b) Each gram of C^{14} has an activity of 12 decays per minute. If a sample of material is found to have 45 decays per hour, approximately how old is it?

12. The human population on Earth doubles roughly every 50 years. There were 6 billion humans on earth in the year 2000.

(a) How many people will there be in the year 3010?

(b) How many people would have to inhabit each square kilometer of the planet for this population to fit on earth? (Take the circumference of the earth to be 40,000 km for the purpose of computing its surface area.)