1. part a

\[
\int_{1}^{\infty} \frac{dx}{\sqrt[3]{x}(x^{\frac{3}{2}} + \sqrt{x})} = \lim_{a \to \infty} \int_{1}^{a} \frac{dx}{\sqrt[3]{x}(x^{\frac{3}{2}} + \sqrt{x})}
\]

Let \( u = \sqrt[3]{x} \Rightarrow x = u^3 \)

\[
dx = 3u^2 \, du
\]

\[
x = 1 \Rightarrow u = 1
\]

\[
x = a \Rightarrow u = \sqrt[3]{a}
\]

\[
\Rightarrow = \lim_{a \to \infty} \int_{1}^{\sqrt[3]{a}} \frac{3u^2 \, du}{u(u^3 + u)} = \lim_{a \to \infty} \int_{1}^{\sqrt[3]{a}} \frac{3u^2}{u^2(u^2 + 1)} \, du
\]

\[
= \lim_{a \to \infty} \left[ 3 \tan^{-1}(u) \right]_{1}^{\sqrt[3]{a}}
\]

\[
= \lim_{a \to \infty} 3 \left[ \tan^{-1}(\sqrt[3]{a}) - \tan^{-1}(1) \right]
\]

\[
= 3 \lim_{a \to \infty} \tan^{-1}(\sqrt[3]{a}) - 3 \left( \tan^{-1}(1) \right)
\]

\[
= 3 \left( \frac{\pi}{2} \right) - 3 \left( \frac{\pi}{4} \right) = 3 \left( \frac{\pi}{4} \right) = \left[ \frac{3 \pi}{4} \right]
\]
Part b
\[ \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{(\cos x)^{\frac{3}{4}}} \, dx \]

Since \( \cos \frac{\pi}{2} = 0 \), it makes the denominator zero, so we're dealing with an improper integral. So,

\[ \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{(\cos x)^{\frac{3}{4}}} \, dx = \lim_{a \to \frac{\pi}{2}^-} \int_{\frac{\pi}{3}}^{a} \frac{\sin x}{(\cos x)^{\frac{3}{4}}} \, dx \]

By substitution, \( u = \cos x \), \( du = -\sin x \, dx \)

\[ u = \frac{\pi}{3} \rightarrow u = \cos \left( \frac{\pi}{3} \right) = \frac{1}{2} \]

\[ u = a \rightarrow u = \cos (a) \]

\[ = \lim_{a \to \frac{\pi}{2}^-} \int_{\frac{1}{2}}^{\frac{1}{2}} \frac{-du}{u^{\frac{3}{4}}} = \lim_{a \to \frac{\pi}{2}^-} \int_{\frac{1}{2}}^{\frac{1}{2}} \frac{-u^{-\frac{3}{4}} \, du}{u^{\frac{3}{4}}} \]

\[ = - \lim_{a \to \frac{\pi}{2}^-} \left[ 4 u^{\frac{1}{4}} \right]_{\frac{1}{2}}^{\frac{1}{2}} = -4 \lim_{a \to \frac{\pi}{2}^-} \left[ \sqrt{\cos a} - \sqrt{\frac{1}{2}} \right] \]

\[ = -4 \left( 0 - \sqrt{\frac{1}{2}} \right) = 4 \sqrt{\frac{1}{2}} \]

Or

\[ 4 \left( \frac{1}{2} \right)^{\frac{1}{4}} \]
2a. \[ \frac{dy}{dx} = \frac{y}{x^2+1} \]

\[ y(1) = 1 \]

**Step 1.** Solve the differential equation:

\[ \frac{dy}{dx} = \frac{y}{x^2+1} \quad \Rightarrow \quad \frac{dy}{y} = \frac{dx}{x^2+1} \quad \Rightarrow \quad \ln y = \tan^{-1}(x) + C \]

**Step 2.** Find C using the initial condition \( y(1) = 1 \):

\[ \ln(1) = \tan^{-1}(1) + C \quad \Rightarrow \quad 0 = \frac{\pi}{4} + C \quad \Rightarrow \quad C = -\frac{\pi}{4} \]

**Step 3.** Explicit formula:

\[ \ln y = \tan^{-1}(x) - \frac{\pi}{4} \quad \Rightarrow \quad \text{taking exponential of both sides:} \]

\[ e^{\ln y} = e^{\tan^{-1}(x) - \frac{\pi}{4}} \quad \Rightarrow \quad y = e^{\tan^{-1}(x) - \frac{\pi}{4}} \]

\[ y = e^{\tan^{-1}(x) - \frac{\pi}{4}} = e^{-\frac{\pi}{4}} e^{\tan^{-1}(x)} \]
Part b \[
\begin{align*}
\frac{dy}{dt} &= ye^y + t \\
y(0) &= 1
\end{align*}
\]

Step 1: Solve the differential equation:
\[
\begin{align*}
\frac{dy}{dt} &= ye^y + t \\
\implies \frac{dy}{e^y + 1} &= \int t \, dt \\
\implies \int \frac{dy}{e^y + 1} &= \frac{t^2}{2} + C
\end{align*}
\]

To solve \( \int \frac{dy}{e^y + 1} \), we use substitution \( u = e^y + 1 \) \( \implies du = e^y \, dy \)
\[
\begin{align*}
\int \frac{dy}{e^y + 1} &= \int \frac{du}{u} = \int \frac{du}{u - 1} = \int \frac{1}{u - 1} \, du + \int \frac{-1}{u} \, du \\
\text{Partial Fractions}
\end{align*}
\]

\[
= \ln|u - 1| + (-\ln|u|) + C = \ln|e^y - 1| - \ln|e^y + 1| + C.
\]

Thus,
\[
\ln\left|\frac{e^y}{e^y + 1}\right| = \frac{t^2}{2} + C \\
ln(a) - ln(b) = \ln\left(\frac{a}{b}\right)
\]

Step 2: Find \( C \) using the initial condition:
\[
\ln\left|\frac{e^y}{e^y + 1}\right| = \frac{0}{2} + C \implies C = \ln\left|\frac{e^0}{e^0 + 1}\right|
\]

We'll leave the final answer in implicit form:
\[
\ln\left(\frac{e^y}{e^y + 1}\right) = \frac{t^2}{2} + \ln\left(\frac{e^0}{e^0 + 1}\right) \implies \frac{e^y}{e^y + 1} = e^{-\frac{t^2}{2} + \ln\left(\frac{e^0}{e^0 + 1}\right)}
\]

so we can remove \( |\ | \) !!!!