Recall: We have the following Integrals:

(I) $\int \sec^2 x \, dx = \tan x + C$. (The anti-derivative of $\sec^2 x$ is $\tan x$. Memorize it!!!)

(II) $\int \ln x \, dx = x \ln x - x + C$.

(III) $\int \tan x \, dx = -\ln(|\cos x|) + C$.

You can use the above formulas, If you get one of them in your computations!!

1. Let $F(x) = x^2 \cdot \int_{-x}^{e^x} \ln(t^2) \, dt$. Find $\frac{d}{dx} F(x)$.

   **Hint:** Use the product rule!

   $\frac{d}{dx} F(x) = \frac{d}{dx} \left( x^2 \cdot \int_{-x}^{e^x} \ln(t^2) \, dt \right)$

   $= \frac{d}{dx} (x^2) \cdot \left( \int_{-x}^{e^x} \ln(t^2) \, dt \right) + x^2 \cdot \frac{d}{dx} \left( \int_{-x}^{e^x} \ln(t^2) \, dt \right)$

   **Note:** you do not need to evaluate the first integral. So, you can leave it as integral form, [If you want to compute it, use the identity $\ln(t^2) = 2\ln(t)$]. To find the derivative of the integral, take a look at examples 1 and 2 in the lecture note 14.

2. Evaluate the following integrals:

   (a) $\int \theta \sec^2(\theta) \, d\theta$

   **Hint:** Use Integration by Part. Follow the ILATE order to find the best choice of $u$.

   (b) $\int \sin x \cos x \ln(\sin x) \, dx$

   **Hint:**
   Step 1: Use Substitution rule with $t = \sin x$.
   Step 2: Simplify the integral after substitution.
   Step 3: Apply the Integration by Part to new Integral.

   **Note:** Do not forget to write the final answer in term of $x$. 
(c) \( \int \frac{\ln(\tan x)}{\sin x \cos x} \, dx \)

**Hint:**
Step 1: Use Substitution rule with \( t = \tan x \).
Step 2: Simplify the integral after substitution.
Step 3: Apply Substitution Rule again to new Integral

**Note:** Do not forget to write the final answer in term of \( x \).

(d) \( \int_0^1 x \tan^{-1}(x^2) \, dx \)

**Hint:**
Step 1: Use Substitution rule with \( t = x^2 \).
Step 2: Simplify the integral after substitution.
Step 3: Apply the Integration by Part to new Integral

**Note:** When you use Substitution rule for Definite Integral, You need to change the Upper and Lower limits!

(e) \( \int t^2 \ln^2(t) \, dt \)

**Hint:** Use Integration by Part with \( u = \ln^2(t) \) and \( dv = t^2 dt \) (Following the ILATE order). Then, you will get another integral. To deal with this integral, you need to use Integration by Part again!