Lecture note 4 (Sep 13, 2017)

- There are more examples/details/explanation in the lecture notes
- Homework 1 posted on Monday (Sep 11, 2017)
- Please solve Examples & practice problems in the worksheets and check your answers with my solution (posted online on our webpage)
- Come to office hours/Labs and ask your questions about Homeworks ...

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4. Trigonometry

We need to decide how to measure angles!!!

4.1: Degrees vs. Radians

- Cut the circle into 360 parts. Each part is one degree and we write $1^\circ$. So, a circle has $360^\circ$:

- There are $360^\circ$ degrees in the circle.

From 1 and 2, we get that:

note: $2\pi = 360^\circ$

- The natural units to use are radians. Radians measure angles by distance traveled. In other word, the radian is defined as:

$$\text{Radian} = \frac{\text{distance traveled}}{\text{Radius}}$$

- Distance traveled for circle is the circumference $= 2\pi r$

So,

$$\text{Radian} = \frac{2\pi r}{r} = 2\pi$$

- There are $2\pi$ radians in the circle.
Example 1: $\frac{\pi}{3}$ radian is ... degree?

(A) $30^\circ$  (B) $45^\circ$  (C) $60^\circ$  (D) $90^\circ$

We know that $2\pi = 360^\circ$. This means that $\pi = \frac{360^\circ}{2} = 180^\circ$. Now, we want to compute $\frac{\pi}{3}$ which is $\frac{180^\circ}{3} = 60^\circ$.

In general, we have the following formula:

angle in Degree = angle in rad $\cdot \frac{180^\circ}{\pi}$ or $\frac{\pi}{180^\circ}$. Degree = Rad

Example 2: Fill out the following table.

<table>
<thead>
<tr>
<th>Degree</th>
<th>0°</th>
<th>30°</th>
<th>60°</th>
<th>45°</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
<th>360°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radian</td>
<td>0</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{3}$</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{2}$</td>
<td>$\pi$</td>
<td>$\frac{3\pi}{2}$</td>
<td>$2\pi$</td>
</tr>
</tbody>
</table>

4.2 Trigonometric Functions

Let's recall the definition of trigonometric functions:

$\sin x = \frac{a}{c}$

$\cos x = \frac{b}{c}$

$\tan x = \frac{a}{b}$

$\frac{\sin x}{\cos x} = \frac{a/c}{b/c} = \frac{a}{b} = \tan x$

$\cos^2 x + \sin^2 x = \left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2 = \frac{a^2 + b^2}{c^2}$

$\frac{a^2 + b^2}{c^2} = 1$

$\tan x = \frac{\sin x}{\cos x}$

$\cos^2 x + \sin^2 x = 1$
Example 3: Find \( l \)

We use the formula for \( \sin x \) for a right triangle: here \( x = \frac{\pi}{6} \), so,

\[
\sin x = \frac{l}{4} \Rightarrow \sin \frac{\pi}{6} = \frac{l}{4} \Rightarrow l = 4 \cdot \sin \frac{\pi}{6}
\]

We need to remember that \( \sin \frac{\pi}{6} = \sin 30^\circ = \frac{1}{2} \). Thus, we have

\[
l = 4 \cdot \sin \frac{\pi}{6} = 4 \cdot \frac{1}{2} = 2 \Rightarrow \boxed{l = 2}
\]

We need the special triangles (for special degrees):

- Consider the following square: \( \left[ \cos \left( \frac{\pi}{4} \right) \& \sin \left( \frac{\pi}{4} \right) \right] \)

Pythagoras: \( c^2 = a^2 + b^2 = 1 + 1 = 2 \Rightarrow c^2 = 2 \Rightarrow c = \sqrt{2} \)

\( \Rightarrow c = \sqrt{2} \) (can not be negative)

Length is positive

\[
\begin{align*}
\sin \left( \frac{\pi}{4} \right) &= \frac{a}{c} = \frac{1}{\sqrt{2}} \\
\cos \left( \frac{\pi}{4} \right) &= \frac{b}{c} = \frac{1}{\sqrt{2}}
\end{align*}
\]

Note:

\[
\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}
\]
4. Consider the following triangle: \([ \sin(\frac{\pi}{6}), \cos(\frac{\pi}{6}), \sin(\frac{\pi}{3}), \cos(\frac{\pi}{3}) ]\)

- Pythagoras: \(c^2 = a^2 + b^2 \Rightarrow 2^2 = 1^2 + b^2 \Rightarrow b^2 = 4 - 1 = 3 \Rightarrow b = \sqrt{3}\)

- Since its length cannot be negative, \(b = \sqrt{3}\)

- \(\sin(\frac{\pi}{6}) = \frac{a}{c} = \frac{1}{2}\), \(\cos(\frac{\pi}{6}) = \frac{b}{c} = \frac{\sqrt{3}}{2}\)

- \(\sin(\frac{\pi}{3}) = \frac{b}{c} = \frac{\sqrt{3}}{2}\), \(\cos(\frac{\pi}{3}) = \frac{a}{c} = \frac{1}{2}\)

Practice Problem: Fill out the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>(\frac{\pi}{6}) or 30°</th>
<th>(\frac{\pi}{4}) or 45°</th>
<th>(\frac{\pi}{3}) or 60°</th>
<th>(\frac{\pi}{2}) or 90°</th>
<th>(\pi) or 180°</th>
<th>(\frac{3\pi}{2}) or 270°</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin x)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{\sqrt{2}}) or (\frac{\sqrt{2}}{2})</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(\cos x)</td>
<td>1</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>(\frac{1}{\sqrt{2}}) or (\frac{\sqrt{2}}{2})</td>
<td>(\frac{1}{2})</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>