Lecture note 32 (Nov 24, 2017)

\[ \int u \, dv = uv - \int v \, du \quad \text{Formula} \]

Last class, we learned about Integration By Parts, and used this method to compute the integrals

\[ \int x e^{x} \frac{dx}{u} \quad \text{and} \quad \int x \cos x \frac{dx}{u} \]

Let's continue by considering the following integral

\[ \int x \ln x \frac{dx}{u} \]

\[ u = x \]
\[ dv = \ln x \frac{dx}{dv} \]

\[ du = dx \]
\[ v = ? \]

\[ \text{No idea} \]

This choice for \( u (u=x) \) does not work here !!!
2. Let's try $u = \ln x$. So, our $dv = x \, dx$.

\[ \int x \ln x \, dx = uv - \int v \, du = \left( \ln x \right)' \left( \frac{1}{2} x^2 \right) - \int \frac{1}{2} x^2 \left( \frac{1}{x} \right) \, dx \]

\[ = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx = \frac{1}{2} x^2 \ln x - \frac{1}{2} \left( \frac{x^2}{2} \right) + C \]

Note: Recall that for $\int \frac{\ln x}{x} \, dx$, we use the substitution $u = \ln x$, while for the integral $\int x \ln x \, dx$ we apply IBP method.

$\int x \ln x \, dx \rightarrow$ IBP

$\int \frac{1}{x} \ln x \, dx \rightarrow$ Substitution
As we mentioned before, in order to apply IBP, we should have the best choice for $u$ and $dv$.

How to choose the best option for $u$?

For this, we are looking for a function that is easier to take derivative.

Consider the following list (functions):

- **L** = Logarithmic functions \((\ln x, \ln 7x, \log x, \ldots)\)
- **A** = Algebraic functions \((1, x, x^2, x^{1/2}, 2x^{-3}, \ldots)\)
- **T** = Trigonometric functions \((\cos x, \sin x, \cos 2x, \ldots)\)
- **E** = Exponential functions \((e^x, e^{-2x}, e^{-x}, \ldots)\)

To find the best choice for $u$, we follow **LATE** order !!!

The remaining part of function in the integral is $dv$ !!!
Let's take a second look at the integrals that we already solved.

\[
\int x e^x \, dx \quad \int x \cos x \, dx \quad \int x \ln x \, dx
\]

LATE \quad LATE \quad LATE

\[\begin{align*}
u &= x & dv &= e^x \, dx \\
u &= x & dv &= \cos x \, dx \\
u &= \ln x & dv &= x \, dx
\end{align*}\]

Example 1: Find \( \int (2x+1)e^{-x} \, dx \)

Let's use LATE order:

\[
\int \frac{(2x+1)}{e^x} \, dx
\]

\[
A \quad E
\]

LATE \quad LATE

\[
\begin{align*}
u &= 2x+1 & dv &= e^{-x} \, dx
\end{align*}\]

Solve this next lecture...!