**Important Note: Substitution Rule**

**Good choice for Substitution Rule include:**

1. functions inside of other functions
2. functions under square roots
3. function in denominators
4. functions where something else is the derivative of your substitution

"General Idea"
Quiz #5:

1. Friday 24th November 2017

2. Only Lecture notes 28 and 29

"Important Example: Finding an anti-derivative using an extra condition." "Easy examples of Substitution rule."
Lecture note 30 (Nov 20, 2017)

- Homework #7 is posted (More Weight compared to HW 1-6)
- Homework #7 due date is Friday Dec 1, 2017
- End of year MAPS data for Math 190:
  "Math attitudes and perceptions survey"
  [link: https://survey.ubc.ca/s/MAPS/Nov2017/]
  Please complete the survey before Dec 1
- Please complete the Math 190 Teaching Evaluation Survey before Dec 4, 2017.

"Bonus Point if you do it"

Substitution Rule:

The best choice for the substitution is a function \( g(u) = u \) such that (at least) the main part of its derivative \( g'(x) \) appears in the integral.

- \( \int 2x \sin(x^2) \, dx \). We take \( u = x^2 \), so,
  \[ du = (x^2)' \, dx = (2x) \, dx \]
  derivative of \( x^2 \) appears in integral
  \[ \int 2x \sin(x^2) \, du \]
  \[ = \frac{1}{2} \sin(x^2) \, d(x^2) \]
  \[ \frac{1}{2} \int \sin(u) \, du \]
  \[ = -\frac{1}{2} \cos(u) + C \]
  \[ = -\frac{1}{2} \cos(x^2) + C \]
\[ \int x^2 e^{x^3+1} \, dx \] 
we take \( u = x^3 + 1 \), so,

\[ du = (x^3+1)' \, dx = (3x^2) \, dx \]

\[ g'(x) \]

\[ \int x^2 e^{x^3+1} \, dx \rightarrow g'(x) \]

\[ \text{s somehow } g'(x) = \frac{1}{3} x^2 \]

\[ \Rightarrow = \int \frac{1}{3} e^u \, du \]

Example 1: Find \( \int \sin x \cos x \, dx \)

(1) \( u = \cos x \rightarrow du = (\cos x)' \, dx = (-\sin x) \, dx \)

(2) \( u = \sin x \rightarrow du = (\sin x)' \, dx = \cos x \, du \)

we go for (2) so,

\[ u = \sin x \]

\[ du = \cos x \, dx \rightarrow dx = \frac{du}{\cos x} \]
we have \( \frac{du}{\cos x} \)

\[
\int \frac{\sin x}{u} \cos x \, du = \int u \cdot \cos x \, \frac{du}{\cos x} \\
= \int u \, du \quad (\text{\# I}) \int u^n \, du = \frac{1}{n+1} u^{n+1} + C \\
= \frac{u^2}{2} + C.
\]

We write the final answer in terms of \( u \):

\[
u = \sin x \quad \Rightarrow \quad \int \sin x \cdot \cos x \, dx = \frac{(\sin x)^2}{2} + C.
\]

Example 2: Find \( \int \frac{x}{\sqrt{2x^2 + 3}} \, dx \).

We let

\[ u = 2x^2 + 3 \quad \text{and we have} \]

\[
du = (2x^2 + 3)' \, dx \quad \rightarrow \quad du = (4x) \, dx
\]

so,

\[
\int \frac{x}{\sqrt{2x^2 + 3}} \, dx = \int \frac{x}{\sqrt{u}} \, \frac{du}{4x} = \int \frac{du}{4\sqrt{u}}
\]
we compute \( \int \frac{1}{4} \frac{1}{\sqrt{u}} \, du \)
\[
= \frac{1}{4} \int \frac{1}{u^{\frac{1}{2}}} \, du = \frac{1}{4} \int u^{-\frac{1}{2}} \, du
\]
\[
= \frac{1}{4} \left( \frac{1}{\frac{1}{2}^{-\frac{1}{2}}} + C \right)
= \frac{1}{4} \left( 2 \cdot u^{\frac{1}{2}} \right) + C = \frac{1}{2} \sqrt{u} + C
\]

we write the final answer in terms of \( u \):

\[
\frac{1}{2} \sqrt{2x^2 + 3} + C \quad (u = 2x^2 + 3)
\]

Examples: Find \( \int \frac{\ln x}{x} \, dx \)

\[
= \int \frac{1}{x} \ln x \, dx
\]

so, we take

\( u = \ln x \), then

\( du = \left( \ln x \right)' \, dx = \frac{1}{x} \, dx \)

\( \Rightarrow du = \frac{1}{x} \, dx \rightarrow xdu = x^1 \, du \rightarrow \)
So,

\[ u = \ln x \quad \& \quad \int dx = x \, du \]

Then,

\[ \int \frac{\ln x}{x} \, dx = \int \frac{u}{x} \frac{dx}{du} = \int u \, du = \frac{u^2}{2} + C \]

(Again **)

we write the final answer in terms of \( x \):

\[ \frac{(\ln x)^2}{2} + C \quad \text{(} u = \ln x \text{)} \]

Example 4: Find \( \int x \sqrt{x+3} \, dx \)

we go for under square root:

\[ u = x + 3 \quad \Rightarrow \quad du = (x+3)' \, dx \]

\[ \Rightarrow \quad du = (1) \, dx \]

\[ \Rightarrow \quad du = dx \]
\[ \int x \sqrt{x+3} \, dx = \int x \sqrt{u} \, du \]

We use relation between \( u \) and \( x \) to get rid of \( x \): In other words,

\[ u = x+3 \implies x = u-3 \]

So, we replace \( x \) by \( u-3 \):

\[ \int x \sqrt{x+3} \, dx = \int (u-3) \sqrt{u} \, du \]

"We solve it next lecture"