Let's start by solving two examples to recall what we learned last time:

**Example 1:** Find \( \int_{-1}^{4} (-2x + 11) \, dx \) where \( f(x) = -2x + 11 \)

Area under \( f(x) = -2x + 11 \) between -1 and 4.

\[
\int_{-1}^{4}(-2x+11) \, dx = \frac{(10)(5)}{2} + 3(5) = 25 + 15 \]

\[
= 40
\]

**Example 2:** If \( \int_{-2}^{3} f(x) \, dx = 5 \) and \( \int_{-2}^{3} g(x) \, dx = -1 \), then compute \( \int_{-2}^{3} (-2f(x) + 3g(x)) \, dx \).

Given \( \int_{-2}^{3} f(x) \, dx + \int_{-2}^{3} g(x) \, dx = 5 - 1 = 4 \)

\[
= -2 \left[ \int_{-2}^{3} f(x) \, dx \right] + 3 \left[ \int_{-2}^{3} g(x) \, dx \right]
\]

\[
= -2 \left[ \frac{5}{2} \right] + 3(-1)
\]

\[
= -10 - 3 = -13
\]
Anti-derivative

Given function  
\[ f(x) \rightarrow f'(x) \]

Find  
\[ f'(x) \rightarrow f(x) \]  
backward

we already know how to deal with \( \text{(I)} \). In order to deal with anti-derivative (\( \text{(II)} \)), we only need to think backward process of taking derivative. (using the derivative rules)

Definition of (Anti-derivative)

A function \( F(x) \) is called an Anti-derivative of \( f(x) \)
if

\[ F'(x) = f(x) \]

For example, we know that \( (x^3)' = 3x^2 \).

So \( F(x) = x^3 \) is an anti-derivative of \( f(x) = 3x^2 \).
4. In general, to find an anti-derivative of a function \( f(x) \), we do the following:

Given \( f(x) \) → we think: derivative of what function is \( f(x) \)

\[ F \] → we use derivative rules and find the anti-derivative

we need to think backward of derivative to find anti-derivative.

**Example 3:** Consider \( f(x) = 2x \), the anti-derivative is we think derivative of what function \( F \) is \( 2x \) ?

\[ \frac{d}{dx} x^2 = 2x \rightarrow F = x^2 \] anti-derivative

**Example 4:** What is the anti-derivative of \( f(x) = \cos x \)?

Derivative of what function \( F(x) \) is \( f(x) = \cos x \).

\[ \sin x = \cos x \rightarrow F(x) = \sin x \] Anti-derivative
Example 5: Find an anti-derivative for the following:

\[ \begin{align*}
\text{Given:} & \quad f(x) = x^3, \quad g(x) = \sin x, \quad h(x) = 1, \quad a(x) = \frac{1}{x}, \quad b(x) = \sqrt{x}, \quad k(x) = C \\
\text{Required:} & \quad \text{an anti-derivative of } f(x) = x^3.
\end{align*} \]

To find the anti-derivative of \( f(x) = x^3 \), we think backward and add 1 to the power. So, we have:

\[ x^{3+1} \implies x^4 \]

Let take the derivative of \( x^4 \):

\[ (x^4)' = 4x^3 \]

So, \( x^4 \) is not an A-D for \( x^3 \). We need to get rid of 4!!!

Consider: \( \frac{x^4}{4} \). Now, you can see:

\[ \left( \frac{x^4}{4} \right)' = \frac{1}{4} (x^4)' = \frac{1}{4} (4x^3) = x^3. \]
(4) \( g(x) = \sin x \) .

\[
F(x) = \sin x
\]

\[
(\cos x)' = -\sin x \quad \Rightarrow \quad \text{so,}
\]

\[
(-\cos x)' = -(\cos x)' = -(-\sin x) = \sin x
\]

\[
F(x) = -\cos x
\]
\[ F(x) = \frac{2}{3} x^{3/2} \]
Finding the all anti-derivative of a function \( f(x) \):

We know that

\[(x^2)' = 2x \rightarrow \text{an anti-derivative of} \quad f(x) = 2x \quad \text{is} \quad F(x) = x^2\]

\[(x^2 - 2)' = 2x \rightarrow \text{an anti-derivative of} \quad f(x) = 2x \quad \text{is} \quad F(x) = x^2 - 2\]

\[(x^2 + 3)' = 2x \rightarrow \text{an anti-derivative of} \quad f(x) = 2x \quad \text{is} \quad F(x) = x^2 + 3\]

In general, for any constant \( c \), we have

\[(x^2 + c)' = (x^2)' + c' = 2x + 0 = 2x\]

So, the function \( F(x) = x^2 + c \) gives all anti-derivative of \( f(x) = 2x \).
Let $F$ be any anti-derivative of $f$. Then, all the anti-derivatives of $f$ have the form $F + c$, where $c$ is an arbitrary constant.

"All of them have the same derivative, which is $f(x)$. So, they have the same slope at each point. So, we only have vertical shift :-)"

General Anti-derivative Formulas

<table>
<thead>
<tr>
<th>Function</th>
<th>Anti-derivative</th>
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</thead>
<tbody>
<tr>
<td>$x^n$</td>
<td>$\frac{1}{n+1} x^{n+1} + c$</td>
</tr>
<tr>
<td>$\sin x$</td>
<td>$-\cos x + c$</td>
</tr>
<tr>
<td>$\cos x$</td>
<td>$\sin x + c$</td>
</tr>
<tr>
<td>$e^x$</td>
<td>$e^x + c$</td>
</tr>
<tr>
<td>$\frac{1}{x}$</td>
<td>$\ln</td>
</tr>
<tr>
<td>$k$</td>
<td>$kx + c$</td>
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</tbody>
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