Lecture 21 (Oct 25, 2017)

- Office hours on Friday Oct 27, 2017 (14:00-17:00 at LSK303)
- Lab 7 + Practice Problem + Solution to Practice are posted online
- Review on Topics in Today's lecture note

YOUR DUTY FOR FRIDAY (Oct 27, 2017):

1. Read *Review on Topics*
2. Solve Practice Problem + its solutions
3. Take a look at/solve (if you have time) Midterm 2015 & Midterm 2016

Then,

**WE** solve Midterm 2015 & 2016 together!

[our Midterm → Lecture notes 1 - 20]
We start by recalling the chain rule:

\[ \text{if } h(x) = f(g(x)), \text{ then } h'(x) = f'(g(x)) \cdot g'(x). \]

Today, we use the chain rule to formulated some special derivative rules:

we first assume that \( u(x) \) is a function of \( x \), that is 
"\( u(x) \) could be any function in terms of \( x \)."

\[ \text{Formula } 1 \]

\[ \left( \left( \frac{u(x)}{x} \right)^n \right)' = n \left( \frac{u(x)}{x} \right)^{n-1} \cdot \frac{u'(x)}{x} \]

\[ \text{Composition of } f(x) = x^n \text{ and } u(x) = g(x) \]

Find \( \left( \frac{2x+1}{x} \right)^5 \):

\[ \left( \frac{2x+1}{x} \right)^5 = 5 \left( \frac{2x+1}{x} \right)^4 \cdot \frac{2x+1}{x} = \frac{5(2x+1)^4}{x} \cdot \frac{2}{x} \]
3. Find \( \left( \frac{-x^3+x}{u(x)} \right)' \)

\[
\left( (-x^3+x)^{-2} \right)' = -2 (-x^3+x)^{-2-1} \cdot \frac{(-x^3+x)'}{u(x)^{n-1} \cdot u'(x)}
\]

\[
= -2 (-x^3+x)^{-3} (-3x^2+1).
\]

\*\* Formula 2 \*\*

\[
(Sin(u(x)))' = Cos(u(x)) \cdot u'(x)
\]

Composition of \( f(x) = \sin x \) (outside) and \( g(x) = u(x) \) (inside).

- Find \( \left( \frac{\sin(e^x)}{u(x)} \right)' \)

\[
(Sin(e^x))' = \cos(e^x) \cdot (e^x)' = \cos(e^x) \cdot e^x
\]

- Find \( \left( \sin(-2x^5-x) \right)' \)

\[
(Sin(-2x^5-x))' = \cos(-2x^5-x) \cdot (-2x^5)' \]

\[
= \cos(-2x^5-x) \cdot (-10x^4-1)
\]
4. **Formula 3**

\[
\left( \cos(u(x)) \right)' = -\sin(u(x)) \cdot u'(x)
\]

Composition of \( f(x) = \cos x \) (outside) and \( g(x) = u(x) \) (inside)

- Find \( \frac{d}{dx} (\cos(u(x)))' \)

\[
(\cos(u(x)))' = -\sin(u(x)) \cdot (u(x))'
\]

\[
= -\sin(u(x)) \cdot \cos x
\]

---

4. **Formula 4**

\[
(e^{u(x)})' = e^{u(x)} \cdot u'(x)
\]

Composition of \( f(x) = e^x \) (outside) and \( g(x) = u(x) \) (inside)

- Find \( \frac{d}{dx} (e^{2x})' \)

\[
(e^{2x})' = (e^{2x}) \cdot (2x)' = 2xe^{2x}
\]

- Find \( \frac{d}{dx} (e^{-x^2})' \)

\[
(e^{-x^2})' = (e^{-x^2}) \cdot (-x^2)' = -2xe^{-x^2}
\]

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4. **Formula 8**

\[
\left( \ln(u(x)) \right)' = \frac{1}{u(x)} \cdot u'(x) = \frac{u'(x)}{u(x)}
\]

Composition of \( f(x) = \ln x \) (outside) and \( g(x) = u(x) \) (inside)

- Find \( \frac{d}{dx} (\ln(2x+3))' \)

\[
\left( \ln(2x+3) \right)' = \frac{1}{2x+3} \cdot (2x+3)' = \frac{1}{2x+3} \cdot 2 = \frac{2}{2x+3}
\]
Let's now use these five new formulas to compute the derivative of a function which is composition of more than two functions. Let's look at the last example in lecture note 20: we want to compute \((\ln(\sqrt{x-2}))')\.

So, we start with the outside function and keep going to get to the inside one (using 5 formulas):

\[
(\ln(\sqrt{x-2}))' = \frac{1}{\sqrt{x-2}} \cdot (\sqrt{x-2})'
\]

\[
= \frac{1}{\sqrt{x-2}} \cdot \left(\frac{1}{2}(x-2)^{-\frac{1}{2}}\right) 
\]

\[
= \frac{1}{\sqrt{x-2}} \cdot \frac{1}{2} \cdot (x-2)^{-\frac{1}{2}} \cdot (x-2)'
\]

\[
= \frac{1}{\sqrt{x-2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x-2}} 
\]

\[
= \frac{1}{2(x-2)}
\]
Example 1: (Midterm 2018)

Find the derivative of \( f(x) = \sqrt{\ln(3x) + e^{\sin x}} \).

We again use these 5 new formulas. We start by outside one, we take derivative to reach the inside one.

So,

\[
f'(x) = \left( \sqrt{\ln(3x) + e^{\sin x}} \right)' = \left( \left( \ln(3x) + e^{\sin x} \right)^{\frac{1}{2}} \right)'
\]

1. \[
\frac{1}{2} \left( \ln(3x) + e^{\sin x} \right)^{-\frac{1}{2}} \cdot \left( \ln(3x) + e^{\sin x} \right)'
\]

2. \[
\ln(3x) \rightarrow \frac{1}{2} \left( \ln(3x) + e^{\sin x} \right)^{-\frac{1}{2}} \left[ \left( \ln(3x) \right)' + \left( e^{\sin x} \right)' \right]
\]

3. \[
e^{-\sin x} \rightarrow \frac{1}{2} \left( \ln(3x) + e^{\sin x} \right)^{-\frac{1}{2}} \left[ \frac{3}{3x} + e^{\sin x} \cdot (\sin x)' \right]
\]

4. \[
= \frac{1}{2} \left( \ln(3x) + e^{\sin x} \right)^{-\frac{1}{2}} \left[ \frac{3}{3x} + e^{\sin x} \cdot (\cos x) \right]
\]

5. \[
= \frac{1}{2} \left( \ln(3x) + e^{\sin x} \right)^{-\frac{1}{2}} \left( \frac{1}{x} + e^{\sin x} \cdot \cos x \right)
\]