Recall that

\[
\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

"definition of derivative"

Last class, we saw that if \( f(x) = x^3 \), then \( f'(x) = 3x^2 \). But, How? We applied ... the definition of the derivative:

\[
\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},
\]

where \( f(x) = x^3 \) and \( f(x+h) = (x+h)^3 \)
So, we have

\[ f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} \]

\[ = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \]

\[ = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} \]

\[ = \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} \]

\[ = \lim_{h \to 0} (3x^2 + 3xh + h^2) \]

\[ = 3x^2 + 0 + 0 \]

\[ = 3x^2 \]

The derivative is a function which gives the slope of \( f(x) \) at each point.
How to find the equation of the tangent line:

Let's consider $f(x) = x^3$. What is the equation of the line tangent to $f(x)$ at $x = 2$?

We use the derivative to answer this question!!!

we first note that at $x = 2$, we have $f(2) = 2^3 = 8$. So, we are at point $(2, 8)$.

In order to write the equation of the tangent line at point $(2, 8)$, we only need the slope ($m_{\text{tan}}$) which is given by the derivative of $f(x) = x^3$ at $x = 2$. (one point + slope) we can write the equation of the line we just saw that if $f(x) = x^3$, we have $f'(x) = 3x^2 \implies$ slope at $x = 2$ is $f'(2) = 3(2)^2 = 12$.

Thus, $m_{\text{tan}} = 12$ and point is $(2, 8)$.

Therefore,

$y - 8 = 12(x - 2) \implies y = 12x - 16 \color{red}{\checkmark}$
Let's collect what we have so far:

\[(x^2)' = 2x\]
\[(x^3)' = 3x^2\]

Any guesses for

\[(x^4)' = 4x^3\]

In general, we have the \textbf{Power Rule}:

\[
\frac{d}{dx} (x^n) = (x^n)' = n x^{n-1} \quad (n \text{ is any real number})
\]

The proof goes like this:

\[
\frac{d}{dx} (x^n) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}
\]

"a bunch of algebra"   =  \ldots \ldots \\
\[= nx^{n-1}\]
Example 1: Use power rule to find the derivative of

(A) \( f(x) = \sqrt{x} \) we can rewrite \( f(x) = \sqrt{x} \) as:
\[
\frac{df}{dx} = f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2 \sqrt{x}}
\]

(B) \( f(x) = \frac{1}{x} \) we write \( f'(x) = \)
\[
\frac{df}{dx} = f'(x) = (-1) x^{-1-1} = -1 x^{-2} = \frac{-1}{x^2}
\]

Example 2: what is the derivative of \( f(x) = 5 \) ?

(A) 0  (B) 5x  (C) 5  (D) \( \frac{1}{5x} \)

Method 1: Graph

\[\text{slope} = 0\]

Method 2: The Power Rule,
\[
f(x) = 5x^0 \quad \Rightarrow \quad f'(x) = 5(0)x^{0-1} = 0\]
Method 3: The definition of the derivative: \( f'(x) = 5 \)

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{5-5}{h} = 0.
\]

**Note:**
- The derivative of any constant is zero.
- \[ \frac{d}{dx} C = (C)' = 0 \] for any constant \( C \).

**Example 2:** Find the derivative. We use the power rule:

(A) \( f(x) = x^{\pi} \) \( \Rightarrow \) \[ \frac{df}{dx} = f'(x) = \pi x^{\pi-1} \]

"\( \pi \) is a constant"

(B) \( f(x) = \frac{1}{x^2} + \sqrt[3]{2} x^{\frac{3}{2}} + 2017 x + e^{\pi} \)

"Note: We can take the derivative of each term and add them together"

\[
\frac{1}{x^2} = x^{-2} \rightarrow (x^{-2})' = -2x^{-3} = -2x = -\frac{2}{x^3},
\]

\[
(x^{\frac{3}{2}})' = \frac{3}{2} x^{\frac{3}{2}-1} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x}.
\]

\[
(x^{2017})' = 2017 x^{2017-1} = 2017 x^{2016}.
\]

\[
(e^{\pi})' = 0 \quad "e^{\pi} \text{ is constant}."
\]