Recall that \( f(x) = x^2 \) \( \rightarrow \) \( m_{\text{tan}} = 2x = f'(x) \)

Last class, we saw that if \( f(m) = x^3 \), then \( f'(m) = 3x^2 \).

But, How? We applied the definition of the derivative:

\[
\frac{df}{dx} = f'(m) = \lim_{h \to 0} \frac{f(m+h) - f(m)}{h},
\]

where \( f(m) = x^3 \) and \( f(m+h) = (x+h)^3 \)
so, we have \( f(x) = x^3 \)

\[
f'(x) = \lim_{{h \to 0}} \frac{{(x+h)^3 - x^3}}{h} \to \frac{0}{0} \]

\[
= \lim_{{h \to 0}} \frac{{x^3 + 3x^2h + 3xh^2 + h^3}}{h}
\]

\[
= \lim_{{h \to 0}} \frac{3x^2h + 3xh^2 + h^3}{h}
\]

\[
= \lim_{{h \to 0}} \frac{h(3x^2 + 3xh + h^2)}{h}
\]

\[
= \lim_{{h \to 0}} (3x^2 + 3xh + h^2)
\]

\[
= 3x^2 + 0 + 0
\]

\[
= 3x^2
\]

The derivative is a function which gives the slope of the tangent line to the function at each point.

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The derivative is a function which gives the slope of the tangent line to the function at each point.
How to find the equation of the tangent line:

Let's consider \( f(x) = x^3 \). What is the equation of the line tangent to \( f(x) \) at \( x = 2 \)?

We use the derivative to answer this question!!!

From last exercise, if \( f(x) = x^3 \) then \( f'(x) = 3x^2 \)

\[
m_{\text{tan}} = f'(x) \quad \xrightarrow{x=2} \quad m_{\text{tan}} = 3 \times 2^2 = 12
\]

A point on the line is the point where \( x^3 \) and the tangent touch each other, \((2, 8)\)

Equation:

\[
y - y_0 = m(x-x_0)
\]

\[
y - 8 = 12(x - 2)
\]

\[
y = 12x - 24 + 8
\]

\[
y = 12x - 16
\]
Recall: We need the slope of the line and a point on the line (can be y-intercept or any other point).

\[ y = mx + b \]

\[ y - y_0 = m(x - x_0) \]
Let's collect what we have so far:

If \( f(x) = x^2 \); \( f(x) = 2x \)

\[ (x^2)' = 2x \]

\[ (x^3)' = 3x^2 \]

Any guesses for

\( (x^4)' = 4x^3 \)

In general, we have the \( \frac{d}{dx} (x^n) = (x^n)' = nx^{n-1} \) for any power \( n : x^2, x^3, x^4, x^{\frac{1}{2}}, x^{-1} \).

The proof goes like this:

\[
\frac{d}{dx} (x^n) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}
\]

algebra = \( n \cdot x^{n-1} \)
Example 1: Use power rule to find the derivative of

(A) \( f(x) = \sqrt{x} = x^{\frac{1}{2}} \)

Power Rule \( \Rightarrow \quad f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} \)

\( f'(4) = \frac{1}{2 \sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4} = \frac{1}{2 \cdot \sqrt{2}} \)

(B) \( f(x) = \frac{1}{x^{1.1}} \)

\( f(x) = x^{-1.1} \)

\( f'(x) = -1.1 x^{-1.1-1} = -x^{-2} = \frac{-1}{x^2} \)

\( f'(-3) = \frac{-1}{(-3)^2} = \frac{-1}{9} \)

Example 2. What is the derivative of \( f(\sqrt{5}) = 5 \) ?

1\(^{\text{st}}\) method

\( \sqrt{5} \)  \( \rightarrow \)  5 \( \rightarrow \) \( f'(x) = 0 = f'(x) \)

2\(^{\text{nd}}\) method

\( f(x) = 5 = 5 x \)

\( f'(x) = 5(0)x^{0-1} = 0 \)
6) \( f(x) \rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

\( f(x) = 5 \)

\[ = \lim_{h \to 0} \frac{5 - 5}{h} \]

\[ = 0 \]

**Note:**
- The derivative of any constant is zero. If \( f(x) = C \)
- \( \frac{d}{dx} C = (C)' = 0 \) for any constant \( C \).

**Example 2:** Find the derivative:

(A) \( f(x) = x^\Pi \)

\( f'(x) = \Pi x^{\Pi-1} \)

(B) \( f(x) = \left(\frac{1}{x^2}\right) + x^{3/2} - x + e^{\Pi} \)

\( f'(x) = -2x^{-3} + \frac{3}{2} x^{3/2-1} - 2017 x^{2017-1} + 0 \)

\[ = -2x^{-3} + \frac{3}{2} x^{1/2} - 2017 x^{2016} \]

\( f(2) = -2(2^{-3}) + \frac{3}{2} \sqrt{2} - 2017(2^{2016}) \)

\[ = -\frac{2^1}{2^3} + \frac{3}{2} \sqrt{2} - 2017 \times 2^{2016} = -\frac{1}{4} + \frac{3\sqrt{2}}{2} - 2017.2 \]