MATH 305:201, 2017W
Midterm 1 — January 31, 2018

Closed book examination Time: 50 minutes

Last Name ____________________________ First Name ____________________________

Student Number ____________________________

Signature ____________________________

Instructions.

You have 50 minutes to solve the following three problems.
In Problem 1, no justification of your answers is needed. A correct answer is 1 point, no answer is 0 point and a wrong answer -1 point.
In Problems 2 and 3, you must carefully justify your answers.
If the space below the problems is not sufficient, use the back of the pages.
Put away books, notes, laptops, cell phones and turn off your cell phones.

Good luck!

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Student Conduct during Examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  (a) speaking or communicating with other candidates, unless otherwise authorized;
  (b) purposely exposing written papers to the view of other candidates or imaging devices;
  (c) purposely viewing the written papers of other candidates;
  (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) — (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).
Problem 1. [8 points]

Decide whether the following statements are true or false. You do not need to justify your answer.

1. \( \frac{2+i}{1-i} = \frac{1}{2} + i \)
   □ True □ False

2. \(|e^{iz}| < 1\) for all \( z \in \mathbb{C} \) such that \(|z - (1 + i)| < 1\)
   □ True □ False

3. The function \( f(z) = e^{iz} \) is periodic along any horizontal line.
   □ True □ False

4. The function \( z \mapsto \frac{1}{z} \) maps any disc in \( \mathbb{C} \) onto a half-plane
   □ True □ False

5. \((\sin(\alpha))^3 = -(1/4) \sin(3\alpha) + (3/4) \sin(\alpha)\) for all \( \alpha \in \mathbb{R} \)
   □ True □ False

6. Assume that \( u, v \) are two real-valued functions defined on \( \mathbb{R}^2 \). For \( f = u + iv \) to be holomorphic in \( \mathbb{C} \), it suffices that \( \Delta u(x, y) = \Delta v(x, y) = 0 \) for all \( (x, y) \in \mathbb{R}^2 \), where \( \Delta = \partial_x^2 + \partial_y^2 \).
   □ True □ False

7. If \( z \mapsto z^{1/2} \) is the principal branch of the square root, then \((-i)^{1/2} = e^{-i\pi/4}\)
   □ True □ False

8. There is a branch of the complex logarithm that is holomorphic in \( \mathbb{C} \setminus \{ z \in \mathbb{C} : \text{Im}(z) = 0 \text{ and } \text{Re}(z) \geq 0 \} \)
   □ True □ False
Problem 2. [5+3 points]

(i) Find all solutions $z \in \mathbb{C}$ of the following equations:

(a) $\sinh(2z) = i$

(b) $z^3 = w$, where $w = \rho e^{i\theta} \in \mathbb{C} \setminus \{0\}$ ($\rho, \theta \in \mathbb{R}$) is arbitrary but fixed.

(ii) Determine the domain where $f(z) = \log(1 - z^3)$ is holomorphic. Sketch the branch cuts.

*Hint:* Use (ib).
Problem 3. [4+4 points]

(i) Explain why the function \( v(x, y) = -x + 3x^2y - y^3 \) can be the imaginary part of a holomorphic function, and find a \( u(x, y) \) such that \( f = u + iv \) is holomorphic.

(ii) Let \( f : \mathbb{C} \to \mathbb{C} \) be entire. Define the function \( g : \mathbb{C} \to \mathbb{C} \) by

\[
g(z) := \overline{f(\overline{z})},
\]

where \( \overline{\cdot} \) denotes complex conjugation. Show that \( g \) is entire and that \( g'(z) = \overline{f'(\overline{z})} \).

Hint: Write \( f = u + iv, \ g = h + ik \) and derive relations between these functions and between their derivatives. Recall also that \( g'(z) = \partial_x h(x, y) + i \partial_y k(x, y) \).