Problem 1. Find all solutions of the following equations:
(i) $\cosh(5z) = 0$
(ii) $2\cos(z) = i\sin(z)$
(iii) $(z - i)^4 = (z + i)^4$

Problem 2. (i) Show that $|\cos(z)|$ tends to $+\infty$ as $|z| \to +\infty$ along a straight line through 0 with non-zero slope
*Hint:* Write $z = re^{i\theta}$ with $\theta \neq n\pi$ fixed and let $r \to \infty$
(ii) Show that $\sin(z)$ is real if and only if either $\text{Im}(z) = 0$ or $\text{Re}(z) = \pi(n + 1/2)$, $n \in \mathbb{Z}$.

Problem 3. Find a (piecewise) smooth parametrization of the following contours:
(i) The curve in the figure to the right
(ii) The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $z = x + iy$), running once from $z_i = a$ to itself with the positive orientation.

Problem 4. Compute
(i) $\int_{\alpha} zdz$
(ii) $\int_{\alpha} \bar{z}dz$
(iii) $\int_{\alpha} \text{Re}(z)dz$
along the polygonal arc $0 \to 1 + i \to 2$.

Problem 5. (i) If $\alpha$ is the straight line from $i$ to $-1$, show without computing the integral that
\[ \left| \int_{\alpha} \frac{1}{z^2}dz \right| \leq 2\sqrt{2}. \]
(ii) Let $k > 0$. Show that
\[ \lim_{R \to +\infty} \left| \int_{\alpha_R} \frac{e^{ikz}}{1 + z^2}dz \right| = 0 \]
where $\alpha_R$ is the semi-circle of radius $R$ in the upper half-plane centred at the origin.
(iii) Show that
\[ \lim_{R \to +\infty} \left| \int_{\gamma_R} e^{-z^2}dz \right| = 0 \]
where $\gamma_R$ is vertical line segment from $R$ to $R + ih$, $h > 0$ fixed.