There are two parts to this assignment. The first part is on WebWorK (accessible through Canvas). The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence of your solutions.

Your solutions must be typed or handwritten clearly, with your name, student number and recitation number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together. Your written assignment must be handed in at the end of your recitation on Friday, September 27. The online assignment will close at 9:00am on Friday, September 27.

**Problem 1.** Let \((a_n)_{n \in \mathbb{N}}\) be a bounded sequence. Define a crest of \((a_n)_{n \in \mathbb{N}}\) as a term \(a_m\) which is greater than all subsequent terms, namely \(a_m > a_n\) for all \(n > m\).

Assume that \((a_n)_{n \in \mathbb{N}}\) has infinitely many crests. Prove that the crests form a convergent sequence.

**Problem 2.** Let \((a_n)_{n \in \mathbb{N}}\) be the sequence defined by
\[
a_1 = 1, \quad a_{n+1} = \frac{1}{3}(a_n + 6) \quad (n \geq 1)
\]

1. Prove that if \(a_{n-1} < a_n\), then \(a_n < a_{n+1}\)

2. Explain in a few sentences why one concludes from \(a_1 < a_2\) and the previous part that \((a_n)_{n \in \mathbb{N}}\) is increasing.

3. Prove that if \(a_n < 6\), then \(a_{n+1} < 6\)

4. Explain in a few sentences why one concludes from \(a_1 < 6\) and the previous part that \((a_n)_{n \in \mathbb{N}}\) is bounded above.

5. Prove that \((a_n)_{n \in \mathbb{N}}\) is convergent.

6. Calculate \(\lim_{n \to \infty} a_n\).

**Problem 3.** This problem will not be graded and does not need to be handed in.

Let \(w > 1\) and \(l \in \mathbb{N}, l \neq 0\). Prove that the sequence \(a_n = \frac{n^l}{w^n}\) converges to 0:

\[
\lim_{n \to \infty} \frac{n^l}{w^n} = 0
\]

**Hint:** Reduce the problem to the case \(l = 1\), then use \(w > 1\) and the binomial formula.