Problem 1. The singularities of the integrand lie at \( \{ n \in \mathbb{Z} \} \). The Taylor series of \( \sin(\pi z) \) at \( z_0 = 0 \) reads 
\[
\pi z - (1/6)(\pi z)^3 + \ldots = \pi z(1 - (1/6)(\pi z)^2 + \ldots)
\]
so that for \( |z| \) sufficiently small, \( z \neq 0 \),
\[
f(z) = \frac{z^2(z-1)}{\sin^2(\pi z)} = \frac{z^2(z-1)}{(\pi z)^2} (1 - (1/6)(\pi z)^2 + \ldots)^{-2} = \frac{z - 1}{\pi^2} (1 + (1/3)(\pi z)^2 + \ldots) = -\frac{1}{\pi^2} + \frac{z}{\pi^2} + \ldots
\]
proving that \( z_0 = 0 \) is a removable singularity. In particular, \( \text{Res}(f; 0) = 0 \). Similarly, \( \sin(\pi z) = \sin(\pi \cdot 1) + \pi \cos(\pi \cdot 1)(z - 1) + \ldots \) so that
\[
f(z) = (z - 1 + 1)^2 \frac{z - 1}{\pi^2(z - 1)^2}(1 + \ldots) = \frac{1}{\pi^2} \frac{2}{\pi^2} + \ldots
\]
Hence \( z_0 = 1 \) is a simple pole with \( \text{Res}(f; 1) = 1/\pi^2 \). Note that the other integers are poles of order two since \( (\sin^2(\pi z))'|_{z=n} = 0 \) while \( (\sin^2(\pi z))''|_{z=n} \neq 0 \). In particular, \( \sin^2(\pi z) = \pi^2(z + 1)^2 + \ldots \), so that
\[
\text{Res}(f; -1) = \lim_{z \to -1} ((z + 1)^2 f(z))' = \frac{1}{\pi^2} \lim_{z \to -1} (z^2(z - 1))' = \frac{5}{\pi^2}.
\]
We conclude \( f|_{z=\frac{1}{2}} = \frac{z^2(z-1)}{\sin^2(\pi z)} = 2\pi i(\frac{1}{\pi^2} + \frac{5}{\pi^2}) = \frac{12i}{\pi} \).

Problem 2. We first note that \( f(z) = \frac{1}{(z-1)(z+i)} = \frac{1}{2} \left( \frac{1}{z+i} - \frac{1}{z-i} \right) \). Now \( |1 - z| < \sqrt{2} = |1 \pm i| \), so that we have the convergent series
\[
\frac{1}{z \pm i} = \frac{1}{(1 \pm i) \left( 1 - \frac{1}{1 \pm i} z \right)} = \frac{1}{(1 \pm i)} \sum_{k=0}^{\infty} (-1)^k \frac{1}{(1 \pm i)^k} (z - 1)^k
\]
We write \( 1 \pm i = \sqrt{2} e^{\pm i \frac{\pi}{4}} \) and take the difference to get
\[
f(z) = \frac{i}{2} \sum_{k=0}^{\infty} (-1)^k \left( \frac{1}{(1+i)^{k+1}} - \frac{1}{(1-i)^{k+1}} \right) (z - 1)^k = \frac{i}{2} \sum_{k=0}^{\infty} (-1)^k \frac{1}{2^{k+1}} (e^{-i(k+1)\frac{\pi}{4}} - e^{(k+1)\frac{\pi}{4}}) (z - 1)^k
\]
\[
= \sum_{k=0}^{\infty} (-1)^k \frac{1}{2^{k+1}} \sin \left( (k+1)\frac{\pi}{4} \right) (z - 1)^k
\]
(iia) In a similar fashion, but expanding only \((z + i)^{-1} \),
\[
f(z) = \frac{1}{z - i} \frac{1}{2i} \left( 1 - \frac{1}{z-i} \frac{1}{2} \right) \frac{1}{z - i} \sum_{k=0}^{\infty} (-1)^k \frac{1}{(2i)^{k+1}} (z - 1)^k = -\sum_{k=-1}^{\infty} \frac{i^{k+2}}{2^{k+2}} (z - i)^k
\]
where we used that \( \frac{1}{z} = i \).

(iib) Finally, writing \( z + i = (z - i)(1 - \frac{2i}{(z-i)}) \), we obtain
\[
f(z) = \frac{1}{(z-i)^2} \sum_{k=0}^{\infty} (-2i)^k (z - 1)^{-k} = -\sum_{k=-\infty}^{-2} \frac{i^k}{2^k} (z - i)^k.
\]
Problem 3. The integrand has one simple zero at $z_0 = 1$ and three simple poles at $z_1 = 0$, $z_2 = -\frac{1}{2}$, $z_3 = 2$. Therefore, the disk of radius $\frac{3}{4}$ contains two simple poles so that $f(C_{\frac{3}{4}})$ winds twice around the origin in the negative direction and hence it is the right curve, with the clockwise orientation. Furthermore, the disk of radius $\frac{3}{2}$ contains two simple poles and one simple zero: $f(C_{\frac{3}{2}})$ winds once around the origin in the negative direction ($-2 + 1$) and hence it is the right curve, which must be oriented clockwise again. The disk of radius $\frac{5}{2}$ containing three poles and one zero, all simple, $f(C_{\frac{5}{2}})$ is as plotted in the figure, negatively orientation.