Problem 1. Compute

(i) \( \int_\alpha z \, dz \),  
(ii) \( \int_\alpha \bar{z} \, dz \),  
(iii) \( \int_\alpha \text{Re}(z) \, dz \),

along the polygonal curve \( 0 \to 1+i \to 2 \).

Problem 2. (i) If \( \alpha \) is the straight line from \( i \) to \( -1 \), show without computing the integral that

\[ \left| \int_\alpha \frac{1}{z^2} \, dz \right| \leq 2\sqrt{2}. \]

(ii) Let \( k > 0 \). Show that

\[ \lim_{R \to +\infty} \left| \int_{\alpha_R} e^{ikz} \, dz \right| = 0 \]

where \( \alpha_R \) is the semi-circle of radius \( R \) centred at the origin.

(iii) Show that

\[ \lim_{R \to +\infty} \left| \int_{\gamma_R} e^{-z^2} \, dz \right| = 0 \]

where \( \gamma_R \) is vertical line segment from \( R \) to \( R + ih \), \( h > 0 \) fixed.

Problem 3. (i) Compute

\[ \int_\alpha \text{Log}(z) \, dz \]

where \( \alpha \) is the straight line from \( 1 \) to \( i \).

(ii) Compute

\[ \int_{\alpha_{\pm}} \bar{z} \, dz \]

where \( \alpha_{+} \), resp. \( \alpha_{-} \), is the semi-circle running from \( 1 \) to \( -1 \) in the upper, resp. lower, half-plane.

Problem 4. Let \( \alpha = a + ib \) be any non-zero complex number, and let \( f : \mathbb{R} \to \mathbb{C} \) be defined by \( f(t) = e^{at} \). Use the fact that \( F(t) = \alpha^{-1}e^{at} \) is an antiderivative of \( f \) to show that

\[ \int e^{at} \cos(bt) \, dt = \frac{e^{at}}{a^2 + b^2} \left( a \cos(bt) + b \sin(bt) \right), \quad \int e^{at} \sin(bt) \, dt = \frac{e^{at}}{a^2 + b^2} \left( a \sin(bt) - b \cos(bt) \right). \]

Conclude that for \( a > 0 \),

\[ \int_0^\infty e^{-at} \cos(bt) \, dt = \frac{a}{a^2 + b^2}, \quad \int_0^\infty e^{-at} \sin(bt) \, dt = \frac{b}{a^2 + b^2}. \]