Problem 1. (i) Show that
\[ \int_0^\infty \frac{1}{x^3+1} \, dx = \frac{2\pi\sqrt{3}}{9} \]
by integrating \( f(z) = \frac{1}{z^3+1} \) along the boundary of the sector \( \{r e^{i\theta} : 0 \leq \theta \leq 2\pi/3, 0 \leq r \leq R\} \) and letting \( R \to \infty \).

(ii) Briefly describe how to generalize the method to compute \( \int_0^\infty \frac{1}{x^n+1} \, dx \) for \( n \geq 2 \) (you don’t need to carry out the computation here).

Problem 2. Let \( 0 < a < 1 \). Show that
\[ \int_0^\infty \frac{\ln(x)}{x^a(1+x)} \, dx = \frac{\pi^2 \cos(a\pi)}{(\sin(a\pi))^2} \]

*Hint:* Use the following generalization of what was obtained in class: \( \int_0^\infty \frac{1}{x^{a+1}} \, dx = \frac{\pi}{\sin(a\pi)} \)

Problem 3. (i) Find the Laurent series of \( f(z) = z^3 \sin\left(\frac{1}{2z}\right) \) in \( |z| > 0 \).

(ii) Determine the singular part of \( f \) in \( C(2;0,1) \) and in \( \hat{C}(0;0,\infty) \).

(iii) Compute
\[ \oint_{|z|=1} z^3 \sin\left(\frac{1}{2z}\right) \, dz \]