1. Related rates

Exercise 1. A spherical balloon is being inflated at a rate of $10\pi \text{ cm}^3/\text{sec}$. But, there is a puncture, releasing the air at a rate of $\frac{1}{4}$ of the volume of the balloon. How fast is the radius of the balloon changing when the diameter is 6 meters?

Exercise 2 (CLP-I Questions). The Ship Argo is 400 miles directly south of Nameless Island\footnote{Here we refer to Nameless Island in the Galápagos. Not to be confused with Nameless Island in Sitka, Alaska.} and is sailing south at 20 miles/hour. The Ship Beagle is 300 miles directly east of Nameless Island and is sailing west at 15 miles/hour. At what rate is the distance between the ships changing?

(b) The same setting, but now the Ship Argo is moving towards the island.
Exercise 3 (Final exam 2010: short answer questions). A vertical cylindrical tank with radius 3 m is being filled with water at a rate of 5 m³/min. How fast is the height of the water increasing?

Find more problems from previous final exams at

http://wiki.ubc.ca/Science:Math_Exam_Resources
2. Newton’s Law of Cooling

Newton’s law of cooling indicates

\[
\frac{dT}{dt} = -k(T - A), \quad k > 0
\]

General solution: \( T = \)

\^The textbook takes \( K < 0 \). Here we write down the minus sign \( (K = -k) \) to emphasize that \( K \) is negative

Exercise 4. The temperature of this cozy classroom is 20\(^\circ\)C. I arrived at 9:20 am, at that time the temperature of my coffee was 93\(^\circ\)C. But one minute later, the temperature dropped to 88\(^\circ\)C.

(a) Write the differential equation satisfied by the temperature \( T(t) \) of the coffee. Set 9:21 am as \( t = 0 \).

(b) Determine the time when the temperature of the coffee is 63\(^\circ\)C

(c) Describe the behaviour of the temperature as \( t \to \infty \)

Exercise 5 (Final 2011). (*) A wealthy man was found murdered in his home at 10pm at night. The temperature of his body was 33\(^\circ\)C, and of the room 21\(^\circ\)C. An hour later the temperature of the body was 31\(^\circ\)C. Assume the body cools after death according to Newton’s Law of Cooling.

(a) Normal boy temperature is 37\(^\circ\)C. When did the murder take place?

(b) Did the murder occur before 9pm? Justify your answer.