1 Introduction

This is a discussion on Exercise 1 in the worksheet for Lecture 12. The exercise is the following.

**Exercise 1.** The position of a particle at time $t$ is given by

$$f(t) = \frac{1}{\pi} \cos(\pi t)$$

When is the particle accelerating? Decelerating?

For the sake of simplification, we will solve this exercise for $t > 0$. This simplification is reasonable, since our graphs are either symmetric or antisymmetric around the origin.

The derivative of the position function is the velocity. Similarly, the derivative of the velocity is the acceleration. The chain rule gives us the following:

$$v(t) = f'(t) = -\sin(\pi t) \quad \text{velocity}$$

$$a(t) = v'(t) = -\pi \cos(\pi t) \quad \text{acceleration}$$

Now, to answer the question we need to interpret what we mean by “acceleration” and “deceleration”. We are going to present two alternatives: the first was given in class and the second is similar to a problem in WeBWorK.

2 Solution presented in class

In class we said:

“the particle accelerates if the acceleration function is positive i.e. $a(t) > 0$”

The function $a(t)$ is (strictly) positive in intervals around the odd numbers. Therefore acceleration is positive at the following union of intervals

$$\left(1 - \frac{1}{2}, 1 + \frac{1}{2}\right) \cup \left(3 - \frac{1}{2}, 3 + \frac{1}{2}\right) \cup \left(5 - \frac{1}{2}, 5 + \frac{1}{2}\right) \ldots$$

We can write down the union of intervals in some elegant ways, as we show below.

$$\left(1 - \frac{1}{2}, 1 + \frac{1}{2}\right) \cup \left(3 - \frac{1}{2}, 3 + \frac{1}{2}\right) \cup \ldots = \bigcup_{n \text{ is odd}} (n - 1/2, n + 1/2)$$

$$= \{t \mid n - 1/2 < t < n + 1/2, \text{ n is odd}\}$$

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1 A function $g$ is symmetric around 0 if $g(x) = g(-x)$ A function $h$ is antisymmetric around 0 if $h(x) = -h(-x)$
Similarly, $a(t)$ is (strictly) negative in intervals around even numbers. Therefore, the acceleration is negative at the following union of intervals

$$\left(2 - \frac{1}{2}, 1 + \frac{1}{2}\right) \cup \left(4 - \frac{1}{2}, 3 + \frac{1}{2}\right) \cup \left(6 - \frac{1}{2}, 5 + \frac{1}{2}\right) \ldots$$

### 3 A different interpretation

Now, we will make the following interpretation

“a particle accelerates if the speed increases”

Now, instead of working with velocity and the derivative of velocity, we need to work with speed and the derivative of the speed.

Figure 1: $a(t) = v'(t) = -\pi \cos(\pi t)$

Figure 2: $v(t) = -\sin(\pi t)$
We found above that the speed is

\[ s(t) = |v(t)| = |\sin(\pi t)| \]

If we look to the graph (figure 3), it is easy to describe when the function is increasing. These are the union of the intervals starting at a integer and positive number, and each interval has length 1.2 Therefore, the speed is increasing in the union of intervals

\[ \left( 0, \frac{1}{2} \right) \cup \left( 1, 1 + \frac{1}{2} \right) \cup \left( 2, 2 + \frac{1}{2} \right) \cup \ldots \]  

Note that the set of times where the particle speeds up (intervals in (2)) is different to the set of times where the acceleration is positive (intervals in (1)). To understand why this happens, we compare the graphs of velocity \( v(t) \) (Figure 2) and speed (Figure 3).

In the interval \( \left( 0, \frac{1}{2} \right) \), velocity is decreasing and hence acceleration is negative. However, speed is increasing in \( \left( 0, \frac{1}{2} \right) \) and hence it is “speeding up” in this interval. The particle is increasing its speed, but in the negative direction.