1 Topics

• Chapter 1
  
  – Section 1.1: Sets, elements, infinite sets, finite sets, when two sets are equal, natural numbers, integers, real numbers, rational numbers, cardinality of a set, empty set, set builder notation (reading and writing it), interval notation, and all notation for sets.
  
  – Section 1.2: Ordered pair, cartesian product, notation for cartesian product, cardinality of a cartesian product, ordered triple, \((A \times B) \times C \neq A \times (B \times C)\). (5, 7)
  
  – Section 1.3: Subset, proper subset, notation for subset, empty set is subset of all sets, number of subsets of a finite set. (9,11)
  
  – Section 1.4: Power set, notation for the power set, cardinality of a power set. (11, 17, 19)
  
  – Section 1.5: Union of sets, intersection of sets, set difference, notation for all these. (3)
  
  – Section 1.6: Complement of a set, universal set, notation for the complement.
  
  – Section 1.7: Venn diagrams, how to draw venn diagrams. (9, 11, draw a diagram when \(A \cap B = \emptyset\) and \(B \subset C\). )
  
  – 1.8: Indexed sets, union of indexed sets (definition and how to calculate), intersection of indexed sets (definition and how to calculate) (7, 9, 13)

• Chapter 2
  
  – Section 2.1: What is a statement, open sentence.
  
  – Section 2.2: Truth values of statements combined with and, or, not.
  
  – Section 2.3: Conditional statement, hypothesis, conclusion, truth values of a conditional statement (implication), different phrasings of a conditional statement as english sentences and determining the hypothesis and conclusion. (any odd number for practice)
  
  – Section 2.4: Biconditional statements, truth values of biconditional statements, different phrasings of biconditional statements as english sentences.
  
  – Section 2.5: Truth tables. (11, )
  
  – Section 2.6: Logical equivalence, DeMorgan’s laws, contrapositive law, commutative laws, distributive laws, associative laws. (11, 13)
  
  – Section 2.7: Universal quantifier, existential quantifier, the effect of the order of quantifiers, determining whether a statement with quantifiers is true or false, reading quantifiers from an english sentence. (5,7,9)
  
  – Section 2.8: A statement in the form \(P(x) \Rightarrow Q(x)\) is understood as \(\forall x, P(x) \Rightarrow Q(x)\)
  
  – Section 2.9: Expressing a statement written in mathematical logic as an english sentence and vise versa. (5,9,12)
Section 2.10: Negating a statement written as an English sentence, negating a statement written with symbolic logic (including quantifiers and implications). (9,11)

Section 2.11: Logical inference like if $P \Rightarrow Q$ is true and $P$ is true then what can we conclude about $Q$? ($Q$ is true). See section for more examples.

• Chapter 4: Direct proof, even/odd integers, same parity, opposite parity, divides, divisor, multiple, prime, composite, greatest common divisor, least common multiple, the division algorithm, how to prove a statement directly, clearly state all assumptions in the proof, how to use cases in a proof. (19, 20, 27)

• Chapter 5: Contrapositive, how to write the contrapositive, how to prove a statement using the contrapositive, integer congruence modulo $n$ ($a \equiv b \pmod{n}$), how integer congruence modulo $n$ relates to the remainder in the division algorithm when you divide by $n$, section 5.3 is about mathematical writing (while you should try to apply all of these to your homework item 4 would be nice to see on exams), knowing when to stop trying to prove a statement directly and switch to trying to prove a statement with the contrapositive. (17, 19, 23, 25)

• Chapter 6: Contradiction, how to prove a statement using contradiction, what is a contradiction, rational number, irrational number, when to try contradiction instead of a direct proof or contrapositive proof. (5, 9, 11, 17, 19)

• Section 7.1: How to prove a biconditional statement. (3, 9, 35)

2 How to study for a proof based exam

The following two steps are the best way to prepare for a proof based exam.

1. Know all the definitions and notation.
   • Generate an example which satisfies the definition.
   • If possible, generate an example which doesn’t satisfy the definition.

2. Practice
   • When you practice focus on understanding proofs, so then you may apply similar techniques on a different problem. Straight memorization of proofs from class or on the homework won’t help.
   • You can do some of the practice problems in the next section.
   • You can redo old homework question, then check your homework solutions or the online solutions. Often you can gain a deeper understanding of a problem the second time or make more sense of it.

3 Practice Problems

Too many? Start with the underlined problems and homework problems. After (and after reviewing the definitions) focus on the section you feel less sure on.

Want more? Please email me.
• Chapter 1
  – Section 1.2: 5, 7
  – Section 1.3: 9, 11
  – Section 1.4: 11, 17, 19
  – Section 1.5: 3
  – Section 1.7: 9, 11, draw a diagram when \(A \cap B = \emptyset\) and \(B \subseteq C\).
  – 1.8: 7, 9, 13

• Chapter 2
  – Section 2.3: For more practice with different phrasings of a conditional statement as
    English sentences and determining the hypothesis and conclusion. (any odd number for
    practice)
  – Section 2.5: 11
  – Section 2.6: 11, 13
  – Section 2.7: 5, 7, 9
  – Section 2.9: 5, 9, 12
  – Section 2.10: 9, 11

• Chapter 4: 19, 20, 27
• Chapter 5: 17, 19, 23, 25
• Chapter 6: 5, 9, 11, 17, 19, 21
• Section 7.1: 3, 9, 35

4 Common Misconceptions

1. There is a difference between absolute value and cardinality though they both use the same
   sign.
   - \(|\{-5\}| = 1\) because \(-5\) is a set with one element.
   - \(|-5| = 5\) because \(-5\) is a number.

2. There is a difference between the symbols \(\in\) and \(\subseteq\). Check out page 13 example 1.3 in the
   book for more examples.
   - \(\{1\} \subseteq \{1, 2, 3\}\) and \(1 \in \{1, 2, 3\}\) are both correct.
   - \(1 \subseteq \{1, 2, 3\}\) is incorrect.

3. An implication, its negation, and its converse are all logically different. Though, an implication
   and its contrapositive are logically equivalent.

4. The negation of “For every positive \(\epsilon\) we have \(|\epsilon + 2| > 2\)” is “There exists a positive \(\epsilon\) where
   \(|\epsilon + 2| \leq 2\)”. The negation is not “There exists an \(\epsilon \leq 0, |\epsilon + 2| \leq 2\)”.

5. The negation of \((P \Rightarrow Q)\) is \((P \text{ and } \sim Q)\). The negation is not \((Q \Rightarrow P)\) which is the converse
   nor is it \((\sim P \Rightarrow \sim Q)\). The truth table will show the difference.
5 Not sure if you are ready?

- As you practice, keep a list of problems that you stumble on so you can try them again later. If this list gets shorter you are on the right track.

- If you are able so solve the problem or produce a proof for problems in a timely manner without the aid of solutions, then you should feel more ready.

- Afraid to get stuck on an exam problem? Make a list of proof strategies you have used. Run through this mental list on the exam. Here are some off the top of my head. (use cases, even or odd, use cases determined by the division algorithm and what the remainder can be, contradiction, contrapositive, direct proof, factor an equation, use an equation to conclude divisors, use facts about the number of divisors of 1 or a prime number)