Homework 8

1. Chapter 8: Question 12
2. Chapter 8: Question 18
3. Chapter 8: Question 20
4. If \( a \in \mathbb{Z} \) is odd, then prove that the set \( S = \{a + m : m \text{ is even}\} \) is the set of all odd numbers.
5. Prove or disprove the statement:

   "\( A - (B - C) = (A - B) - C. \)"

6. Prove or disprove the statement:

   "There exists \( c \in \mathbb{R} \) such that the equation \( 2^x = c \) has two solutions."

7. Prove or disprove the statement:

   "There exists \( x \in \mathbb{R} \) such that the number \( f(x) = x^2 + 5x + 4 \) is prime."

8. For any \( x \in \mathbb{R} \), consider the number \( \lfloor x \rfloor \), which is defined to be the greatest integer \( n \) such that \( n \leq x \). For example, \( \lfloor 1 \rfloor = 1 \), whereas \( \lfloor 2.7 \rfloor = 2 \). Given the definition, prove or disprove the statement:

   "For all \( x, y \in \mathbb{R} \), \( \lfloor x + y \rfloor \leq \lfloor x \rfloor + \lfloor y \rfloor. \)"

9. Let \( a, b \in \mathbb{Z} \) and \( n \in \mathbb{N} \). We say that \( b \) is a multiplicative inverse of \( a \) modulo \( n \) if \( ab \equiv 1 \) (mod \( n \)).

   If we have two numbers \( a, b \in \mathbb{Z} \), we say that \( n \in \mathbb{Z} \) is a multiplicative inverse of \( a \) modulo \( b \) if \( a \cdot n \equiv 1 \) (mod \( b \)).

   Given the definition, prove or disprove the statements:

   • Every integer has a multiplicative inverse modulo 17.
   • Every integer has a multiplicative inverse modulo 18.

   (Hint: Proposition 7.1 and the following proposition in Chapter 7.3 would be useful.)