ASSIGNMENT 3
DUE: MONDAY JUNE 14

Heat equation II

Problem 1: Consider the following linear parabolic (heat conduction) boundary value problem:

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 6(\pi - x) \text{ for } 0 < x < \pi \text{ and } t > 0, \]

Initial condition \[ u(x, 0) = x^3 - 3\pi x^2 \text{ for } 0 < x < \pi, \]
Boundary condition \[ u(0, t) = 0, u(\pi, t) = 0 \text{ for all } t \geq 0. \]

(i) Determine the steady state solution \( u_\infty(x) \).

(ii) Let \[ u(x, t) = v(x, t) + u_\infty(x) \]

Identify the PDE, the initial condition and the boundary conditions satisfied by \( v(x, t) \).

(iii) Solve the boundary value problem for \( v(x, t) \) and from this determine the solution \( u(x, t) \).

Problem 2: Solve the following heat conduction boundary value problem:

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 1 - xe^{-t} \text{ for } 0 < x < 1 \text{ and } t > 0, \]

Initial condition \[ u(x, 0) = x \text{ for } 0 < x < 1, \]
Boundary condition \[ \frac{\partial u}{\partial x}(0, t) = e^{-t}, u(1, t) = t \text{ for all } t \geq 0. \]
Problem 3: Solve the following heat conduction boundary value problem:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u + e^{-t} \sin(x)$$

for $0 < x < \pi/2$ and $t > 0$,

Initial condition $u(x, 0) = x$ for $0 < x < \pi/2$,

Boundary condition $u(0, t) = 0, \frac{\partial u}{\partial x}(\pi/2, t) = e^{-t}$ for all $t \geq 0$. 