Problem 1: Find the general solutions of the following ODEs:

(a) $6y'' + y' - 2y = 0$.
(b) $y'' - 3y' + 2y = 4e^x$.
(c) $x^2 y'' - 2xy' - 4y = 0$.
(d) $9x^2 y'' + 3xy' + y = 0$.
(e) $x^2 y'' + 3xy' + 2y = 0$, with $y(1) = 1, y'(1) = -2$.

Problem 2: Find power series solutions about $x_0 = 0$ of the following ODEs:

(a) $y'' + xy' + y = 0$.
(b) $(x^2 + 1)y'' + \frac{7}{2}xy' + y = 0$.

Problem 3: Determine the power series solution of the following initial value problems:

(a) $y'' - xy' - y = 0$ with $y(0) = 1, y'(0) = 0$.
(b) $(x - 1)y'' - (x - 3)y' - y = 0$ with $y(0) = 3, y'(0) = 3$.

Problem 4: Consider the ODE:

$$y'' - 2xy' + \lambda y = 0.$$  

where $\lambda$ is a constant.

(a) Using the series expansion about $x_0 = 0$, determine two independent solutions of the given ODE. Calculate the first four non-zero terms in each case.
(b) Find a polynomial solution when $\lambda = 0, 2, 8$ and 10.

Problem 5: For the following ODEs, find a lower bound for the radius of convergence of the series solution about the point $x_0$.

(a) $(x^2 - 3)y'' + y' + \tan(x)y = 0$ about $x_0 = 0$.
(b) $(4x^2 + 9)y'' + x^4y' + y = 0$ about $x_0 = 2$.

Problem 6: Find the singular points of the following ODEs. For each singular point determine whether they are regular or irregular. For each regular singular point, determine the indicial equation and the exponents at the singularity.

(a) $(x^2 - 4)^2 y'' + 8(x + 2)y' + y = 0$.
(b) $-2(x - 1)^2(x^2 - 2)^3 y'' + 2(x^2 - 1)y' - 4xy = 0$.
(c) $2\tan^2(x) y'' - \sin(x)y' - y = 0$.  

Problem 7: Consider the ODE:
\[ 6x^2(1 + x)y'' + 5xy' - y = 0. \]
(a) Determine the ordinary points, the regular singular points and irregular singular points in \([0, +\infty)\).
(b) Determine two values of \(r\) such that there are solutions to the given ODE of the form \(y(x) = \sum_{n=0}^{+\infty} a_n x^{n+r}\).
(c) Use the series expansion in (b) to determine two independent solutions of the given ODE. Calculate only the first three non-zero terms in each case.

Problem 8: Show that \(x = 0\) is a regular singular point of the following ODE.
\[ 8x^2 y'' - 2xy' + 3(x+1)y = 0. \]
Calculate the first three non-zero terms in each of the two independent solutions about \(x_0 = 0\) of the given ODE.