The statements can contain variables, but not every sentence that contains variables are statements.

ex: P: "If an integer x is a multiple of 6, then x is even" is a true statement whereas

Q: The integer x is even" is not a statement since the "truth value" of this sentence depends on what "x" is.

A sentence, whose truth ("truth value") depends on the value of one or more variables, is called an "open sentence".

2.2 And, or, not:

We can use the words "and" and "or" to combine two statements to form a new statement.

ex: P: The number 8 is even and a power of 2.

This can be seen as the combination of the two statements

R: The number 8 is even

Q: 8 is a power of 2. With the word "and". Since we are using letters to denote sentences, we are also going to introduce new
notation to replace "and", "or" and "not".
To denote \( P \land Q \) we write \( R \land Q \). Here "\( \land \)" denote the word "and".
We see that the statement \( R \land Q \) is 
true if both \( R \) and \( Q \) are true,
otherwise it is false. We can write down 
all the possible "truth values" of this 
statement in what is called, a "truth table".

\[
\begin{array}{ccc}
P & Q & R \land Q \\
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]

In this table, 
\( T \) stands for "True", \( F \) stands for "False".

We can also combine two statements with "or".

ex: The statement 

\( P \): "The number 7 is prime or 18 is odd"

is the combination of the statements 

\( Q \): The number 7 is prime

\( R \): The number 18 is odd.

with the word "or".
We will denote the statements of the form $Q \lor R$ as $Q \lor R$, i.e. $\lor$ stands for "or". Here, when we say $Q \lor R$, we mean "one or both of $Q$ and $R$. We see that such a statement can only be false when both $Q$ and $R$ are false, i.e., if we put it in a truth table, we get

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

If we ever need to express the fact that only one of $Q$ and $R$ is true, we use one of the following "$Q$ or $R$, but not both"

"Either $Q$ or $R$"

"Exactly one of $Q$ or $R$".

Moreover, given any statement $P$, we can form a new statement "it is not true that $P$". For example if we have the statement "$2 \in \emptyset" we can form "it is not true that $2 \in \emptyset"", or from "$2$ is even", we can form "it is not true that $2$ is even" (which, by the way, is a false statement, where we see that the statement "$2$ is even" is true)
We use the symbol \( \neg P \) (or \( !P \) or \( \overline{P} \)) to denote "It's not true that", so \( \neg P \) (or \( !P \) or \( \overline{P} \)) means "It is not true that \( P \)", which we often read as "not \( P \)."

And the truth table for \( \neg P \) is as follows:

<table>
<thead>
<tr>
<th>( P )</th>
<th>( \neg P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

The statement \( \neg P \) is called the negation of \( P \).

### 2.3 Conditional Statements

Given any two statements \( P \) and \( Q \), we can form a new statement \( P \) as "If \( P \) then \( Q \)". This is written symbolically as \( P \Rightarrow Q \) which reads as "If \( P \) then \( Q \)" or " \( P \) implies \( Q \)." We say that this statement is false if it fails to deliver its claim, namely, \( P \Rightarrow Q \) is false when \( P \) is true but yet \( Q \) is false.

Such statements are called conditional statements. Their truth table is as follows:

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \Rightarrow Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Here we see that when \( P \) is false, then \( P \Rightarrow Q \) is automatically true. The reason is that when \( P \) is false,