(1) By F.T.C. : \( F(x) = \sqrt{4 + 2x + \sin x} \), So
\[ F'(1) = \sqrt{6 + \sin 1} \]

(2) We have \( G(x) = F(\ln x) = \int_{\ln e}^{\ln x} \sqrt{4 + 2t + \sin t} \, dt \)
So \( G'(x) = (\ln x)' \cdot F'(\ln x) \)
In problem 1, we saw that \( F'(x) = \sqrt{4 + 2x + \sin x} \), So
\[ F'(\ln x) = \sqrt{4 + 2\ln x + \sin(\ln x)} \]
and then
\[ G'(x) = \frac{1}{x} \cdot \sqrt{4 + 2\ln x + \sin(\ln x)} \]
\[ \Rightarrow G'(1) = 1 \cdot \sqrt{4 + 2\ln 1 + \sin(\ln 1)} = \sqrt{4 + 2 \cdot 0 + \sin(0)} = 2 \]

(3) One can write
\[ H(x) = \int_{\ln e}^{\ln x} \sqrt{4 + 2t + \sin t} \, dt \]
\[ = \int_{1}^{\ln x} \sqrt{4 + 2t + \sin t} \, dt + \int_{1}^{\ln x} \sqrt{4 + 2t + \sin t} \, dt \]
\[ = - \int_{1}^{\ln x} \sqrt{4 + 2t + \sin t} \, dt + \int_{1}^{\ln x} \sqrt{4 + 2t + \sin t} \, dt \]
\[ = - F(2x) + F(x) \]
\[ = F(2x) - F(x) \]
So \( H'(x) = (2x)' \cdot F(2x) - F(x) \)
\[ = 2 \cdot F(2x) - F(x) \]
\[ = 2 \sqrt{4 + 2(2x) + \sin(2x)} - \sqrt{4 + 2x + \sin x} \]

(4) We notice that \( \frac{1}{2} (x^2 + 2x) = 2(x+1) \). Seeing \((x+1)\) being multiplied to the other factor \((x^2+2x)\), suggests to use the substitution \( u = x^2 + x \), then
\[ du = (2x + 1) \, dx \] and \( x = 1 \Rightarrow u = 1; x = -1 \Rightarrow u = -1 \)
\[ \int_{1}^{3} (x+1)(x^2 + 2x)^{\frac{3}{2}} \, dx = \int_{-1}^{3} (x+1)u^{\frac{3}{2}} \, du = \frac{1}{2} \int_{-1}^{3} u^{\frac{5}{2}} \, du = \frac{1}{2} \left( \frac{1}{\frac{5}{2}} u^{\frac{5}{2}} \right)_{-1}^{3} = \frac{1}{14} (3^3 + 1) \]

(4) This question had a typo: the limit of integration had to be \( \int_{1}^{3} \).
\[\int \frac{x^3 + x}{x^3 - x} \, dx = 1 \quad \text{(This question had a typo)}\]

First, we factorize \(x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)\).

Now, we see that the fraction \(\frac{x^3 + x}{x^3 - x}\) simplifies. It is very helpful to simplify it before computing the integration:

\[
\frac{x^2 + x}{x^3 - x} = \frac{x(x + 1)}{x(x - 1)(x + 1)} = \frac{1}{x - 1}
\]

\[\Rightarrow I = \int \frac{1}{x - 1} \, dx = \ln|x - 1| + C\]

(6) We have

\[\int \frac{1}{x^6 + 4 - x^2} \, dx = -\frac{\sqrt{4 - x^2}}{4x} + C\]  

(see solutions of midterm 2 in 2014)

So

\[\int_{1}^{2} \frac{1}{x^6 + 4 - x^2} \, dx = \frac{\sqrt{3}}{4}\]

(7) \(u = x \Rightarrow \begin{cases} u = x \\ v = \sin x \Rightarrow \begin{cases} v = -\cos x \\
\int_{0}^{\pi} x \sin x \, dx = (x \cos x)_{\pi} - \int_{0}^{\pi} (-\cos x) \, dx \\
= (-\pi \cos(\pi)) - (0) + (\sin x)_{\pi} \\
= \pi + (\sin \pi) - (\sin 0) = \pi
\end{cases} \end{cases}\]

\[u(0) = \sin^2(2x) = \frac{1 - \cos(4x)}{2}\]

\[= \int \sin^2(2x) \, dx = \int \frac{1 - \cos(4x)}{2} \, dx = \frac{x}{2} - \frac{\sin 4x}{8} + C\]