1. Let \( u = \langle 1, -1, 0 \rangle \) and \( v = \langle -1, 0, 1 \rangle \). What is \(|(u \cdot v)u + 2v|\)?
   a) \( \sqrt{6} \)  
   b) \( \sqrt{14} \)  
   c) \( 3\sqrt{2} \)  
   d) \( \sqrt{2} \)

2. What is the distance of the point \((4, -1, -3)\) from the plane \( y = 1 \)?
   a) 5  
   b) 4  
   c) 3  
   d) 2

3. Let \( P : x + y = z \) and \( Q : -x + 2c(y + 1) + z = 5 \) be two planes. Find \( c \) for which these two planes are orthogonal.
   a) 0  
   b) 1  
   c) \(-1\)  
   d) 2

4. What is the domain of the function \( z = \ln(4 - x^2 - y^2) \)?
   a) \( \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 4\} \)  
   b) \( \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 4\} \)  
   c) \( \{(x, y) \in \mathbb{R}^2 : 4 - x^2 - y^2 \geq 1\} \)  
   d) \( \{(x, y) \in \mathbb{R}^2 \} \)

5. What is the domain of the function \( z = \sqrt{x + y + \frac{1}{\sqrt{3 - x - y}}} \)?
   a) \( \{(x, y) \in \mathbb{R}^2 : 0 \leq x + y \leq 3\} \)  
   b) \( \{(x, y) \in \mathbb{R}^2 : x + y \geq 0\} \cup \{(x, y) \in \mathbb{R}^2 : x + y < 3\} \)  
   c) \( \{(x, y) \in \mathbb{R}^2 : 0 \leq x + y < 3\} \)  
   d) \( \{(x, y) \in \mathbb{R}^2 : x + y \geq 0 \text{ and } x + y \neq 3\} \)

6. Let \( P : 2x + my - 3z = 50 \) and \( Q : -nx + z = 2 - y \) be two parallel planes where \( m \) and \( n \) are two real numbers. What is \( m + n \)?
   a) \( \frac{1}{3} \)  
   b) \( -\frac{1}{3} \)  
   c) \( -\frac{7}{3} \)  
   d) \( \frac{11}{3} \)

7. What is the angle between vectors \( u = 2i - 3k \) and \( v = i + j + 2k \)?
   a) \( \cos^{-1} \frac{4}{\sqrt{78}} \)  
   b) \( \cos^{-1} \frac{-4}{\sqrt{78}} \)  
   c) \( \cos^{-1} -\frac{1}{\sqrt{39}} \)  
   d) \( \cos^{-1} \frac{4}{\sqrt{39}} \)

8. Which is the equation of a plane passing through the point \( P(2, 3, 0) \) and parallel to the plane \( x + 3z - 4 = 2(y - 1) \)?
   a) \( 2x - y = 1 \)  
   b) \( x - 2y + 3z = 2 \)  
   c) \( x + 2y + 3z = 10 \)  
   d) \( -x + 2y - 3z = 4 \)

9. What is the shape of the level curves of \( z = f(x, y) = -x^2 + 2 + 5y \)?
   a) lines  
   b) circles  
   c) ellipses  
   d) parabolas

10. Let \( Q \) be a plane that is through the point \( P(-1, -2, 3) \) and is perpendicular to the line that connects the point \( P \) and the origin. Which point lies on \( Q \)?
    a) \( (1, 0, -1) \)  
    b) \( (-4, -5, 0) \)  
    c) \( (-2, -4, 6) \)  
    d) \( (1, 2, -3) \)
Solutions:

Problem 1: (b) First we compute \( u \cdot v = (1)(-1) + (-1)(0) + (0)(1) = -1 \). So
\[
(u \cdot v)u + 2v = -u + 2v = (-1, 1, 0) + (-2, 0, 2) = (-3, 1, 2).
\]
Thus \(|(u \cdot v)u + 2v| = |(-3, 1, 2)| = \sqrt{(-3)^2 + 1^2 + 2^2} = \sqrt{14}.

Problem 2: (d) If you imagine or just plot this point in \( xyz \)-coordinate system, you can easily see that the distance is 2. (The closest point on the plane \( y = 1 \) to the point \((4, -1, -3)\) is \((4, 1, -3)\))

Problem 3: (b) First we need to write the planes in the following form:
\[
P : x + y - z = 0,
Q : -x + 2cy + z = 5 - 2c.
\]
Now we see that normal vectors of these two planes are \( n_P = \langle 1, 1, -1 \rangle \) and \( n_Q = \langle -1, 2c, 1 \rangle \). We know that two planes are orthogonal if and only if their normal vectors are orthogonal. This last condition is satisfied if (and only if)
\[
n_P \cdot n_Q = (1)(-1) + (1)(2c) + (-1)(1) = 2c - 2 = 0
\]
So \( c = 1 \).

Problem 4: (a) Let \( g(x, y) \) be any function. The the domain of \( z = \ln(g(x, y)) \) can be computed by the condition \( g(x, y) > 0 \). (Remember that logarithms of negative numbers are not defined) So we require \( 4 - x^2 - y^2 > 0 \) which can be also written as \( x^2 + y^2 < 4 \).

Problem 5: (c) Concerning the first term \( \sqrt{x + y} \), we require \( x + y \geq 0 \). As for the second square root, we ask \( x + y \leq 3 \). But since the term \( \sqrt{3 - x - y} \) is in the denominator, it cannot be zero, which means \( 3 - x - y \neq 0 \). Putting all these together gives \( 0 \leq x + y < 3 \).

Problem 6: (c) The normal vectors of these two planes are \( n_P = \langle 2, m, -3 \rangle \) and \( n_Q = \langle -n, 1, 1 \rangle \). Two planes are parallel if (and only if) their normal vectors are
parallel. Moreover, two vectors are parallel if one is a scalar multiple of the other. (in other words, if $n_P = \alpha n_1$ for some real number $\alpha$) By looking at the third coordinates of these vectors, we can see that $n_P = -3n_Q$. So $m = (-3) \times 1 = -3$ and $2 = (-n) \times (-3) = 3n$ and so $n = \frac{2}{3}$.

Problem 7: (b)

$$
\cos(\theta) = \frac{u \cdot v}{||u|| ||v||} = \frac{-4}{\sqrt{13} \sqrt{6}} = \frac{-4}{\sqrt{78}}.
$$

Problem 8: (d) Normal vector of the plane we are looking for can be taken to be the normal vector of the given plane which is $\langle 1, -2, 3 \rangle$. So the equation will look like $x - 2y + 3z = d$ where $d$ can be found by plugging the point $P(2, 3, 0)$ on the left side: $2 - 6 + 0 = d$. So the equation is $x - 2y + 3z = -4$ or $-x + 2y - 3z = 4$.

Problem 9: (d) The level curves are $-x^2 + 2 + 5y = z_0$ which are parabolas.

Problem 10: (b) The vector connecting the origin to $P$ is $\langle -1, 2, -3 \rangle$. This vector is the normal vector for our plane. So the equation of the plane will be $-x - 2y + 3z = 14$. The only point among the choices lying on this plane is then $(-4, -5, 0)$. 