Name:
Student number:

Time: 10 minutes

Note: Each wrong answer has $\frac{1}{3}$ negative score and each right answer has 1 positive score. If you are not sure about a question, leave it blank.

1. Let $P = (5, 11, 12)$ and $Q = (1, 14, 13)$ be two points. Express $\overrightarrow{PQ}$ in terms of $i = (1, 0, 0)$, $j = (0, 1, 0)$ and $k = (0, 0, 1)$:
   a) $-4i + 3j + k$
   b) $4i + 3j + k$
   c) $4i - 3j - k$
   d) $-4i - 3j - k$

2. Which of the following points is closer to the origin?
   a) $(1, 1, -1)$
   b) $(1, 1, 1)$
   c) $(2, 0, 0)$
   d) $(1, 0, 1)$

3. Assume $u = \langle 4, -3, 0 \rangle$ and $v = \langle 0, 1, 1 \rangle$. What is $4u - v$?
   a) $\langle 16, -11, 1 \rangle$
   b) $\langle 16, -13, -1 \rangle$
   c) $\langle 16, -11, -1 \rangle$
   d) $\langle 4, -1, 0 \rangle$

4. Let $u = 2i - 3k$ and $v = i + 4j + 2k$. What is $u \cdot v$?
   a) $-1$
   b) $-10$
   c) $-4$
   d) $\langle 2, 0, -1 \rangle$

5. Let $u, v$ and $w$ be three vectors such that $u \cdot v = \frac{2}{3}$ and $u \cdot w = -\frac{1}{2}$. Compute $(2v - w) \cdot u$:
   a) $\frac{5}{3}$
   b) $-\frac{5}{3}$
   c) $\frac{5}{6}$
   d) $\frac{11}{6}$

6. What is the angle between vectors $u = i + j$ and $v = 3i - 3j$?
   a) $\frac{\pi}{3}$
   b) $\frac{\pi}{4}$
   c) $\frac{\pi}{3}$
   d) $\frac{\pi}{2}$

7. Find $\alpha$ such that the vector $u = \langle 1, 2, \alpha + 1 \rangle$ is orthogonal to $v = \langle -3\alpha, -5, 5 \rangle$?
   a) $\frac{2}{3}$
   b) $\frac{5}{2}$
   c) $-1$
   d) $\frac{5}{8}$

1. A B C D
2. A B C D
3. A B C D
4. A B C D
5. A B C D
6. A B C D
7. A B C D
8. A B C D
9. A B C D
10. A B C D
Solutions:

Problem 1: (a) We have $\overrightarrow{PQ} = Q - P = \langle -1 - 5, 14 - 11, 13 - 12 \rangle = \langle -4, 3, 1 \rangle = -4i + 3j + k$.

Problem 2: (d) The distance of $(1, 1, -1)$ from the origin $= \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$.

The distance of $(1, 1, 1)$ from the origin $= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$.

The distance of $(2, 0, 0)$ from the origin $= \sqrt{2^2 + 0^2 + 0^2} = 2$.

The distance of $(1, 0, 1)$ from the origin $= \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2} \sqrt{2}$.

Since $\sqrt{2} < \sqrt{3} < 2$, the solution is (d).

Problem 3: (b)

$4u - v = 4 \langle 4, -3, 0 \rangle - \langle 0, 1, 1 \rangle = \langle 16, -12, 0 \rangle + \langle 0, -1, -1 \rangle = \langle 16, -13, -1 \rangle$.

Problem 4: (c) $u.v = \langle 2, 0, -3 \rangle \cdot \langle 1, 4, 2 \rangle = 2 + 0 - 6 = -4$

Problem 5: (d) $(2v - w).u = 2(u.v) - u.w = 2(v.u) - w.u = 2\left(\frac{2}{3}\right) - \frac{-1}{2} = \frac{11}{6}$.

Problem 6: (d) First we compute:

$u.v = \langle 1, 1 \rangle \cdot \langle 3, -3 \rangle = 3 - 3 = 0$. So the two vectors are orthogonal with respect to each other and the angle between them is $\frac{\pi}{2}$.

Problem 7: (b) The two vectors $u$ and $v$ are orthogonal if (and only if) $u.v = 0$.

Let’s compute then:

$u.v = \langle 1, 2, \alpha + 1 \rangle \cdot \langle -3\alpha, -5, 5 \rangle = -3\alpha - 10 + 5(\alpha + 1) = 2\alpha - 5$.

So $u.v = 2\alpha - 5 = 0$ gives $\alpha = \frac{5}{2}$. 
