MATH 200:921, Quiz 5

First Name: __________________________  Last Name: __________________________

Student-No: __________________________

Grade: __________________________

- Do not turn the page until instructed to do so.
- This test is closed book. No calculators or formula sheet allowed.
- You have 20 minutes to write this quiz.
- There are three questions in this quiz, worth a total of 20 points.
Short answer question

1. 4 marks For each of the following statements write $T$ for true or $F$ for false next to it.

1. We always have
\[ \int_{0}^{1} \int_{a(x)}^{b(x)} h(x)g(y) \, dy \, dx = \left( \int_{0}^{1} h(x) \, dx \right) \left( \int_{a(0)}^{b(1)} g(y) \, dy \right). \]

2. If $f(x,y)$ is continuous then it is always true that
\[ \int_{c}^{d} \int_{a}^{b} f(x,y) \, dx \, dy = \int_{a}^{b} \int_{c}^{d} f(x,y) \, dy \, dx. \]

3. We have
\[ \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} x \, dy \, dx = \int_{0}^{\pi} \int_{0}^{1} r^2 \cos(\theta) \, dr \, d\theta. \]

4. If the density function is constant, the center of mass of a region $D$ must always lie inside of $D$. 

Long answer question—you must show your work

2. [8 marks] Consider the integral

\[ \int_0^4 \int_{\sqrt{4-x}}^{\sqrt{4-x}} f(x, y) \, dy \, dx \]

1. Sketch the domain of integration and rewrite the integral as a dx dy integral.
2. Evaluate the integral when \( f(x, y) = e^{8y - \frac{2}{3}y^3} \).
Long answer question—you must show your work

3. **8 marks** Consider the triangle $T$ with vertices $A = (0, 0)$, $B = (1, 1)$, $C = (1, 0)$.

1. Sketch $T$ and describe the side $CB$ with polar coordinates equations $r = a(\theta), c \leq \theta \leq d$.

2. Using an integral in polar coordinates, compute the area of the triangle.

3. Assuming that the mass distribution on $T$ is a constant $\rho$, write integrals in polar coordinates that compute the coordinates of the center of mass of $T$. You do not need to evaluate them.