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Knots and the Jones Polynomial.

Def'n a knot is a closed curve in $\mathbb{R}^3$ that doesn intersect itself.

deformations don't change the knot.

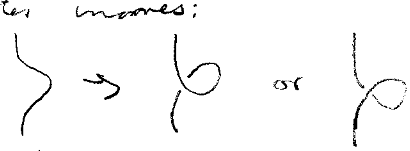
eg unknot  trefoil knot 

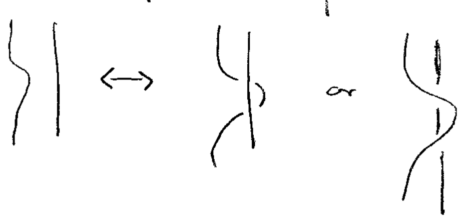
How do we know these are different?

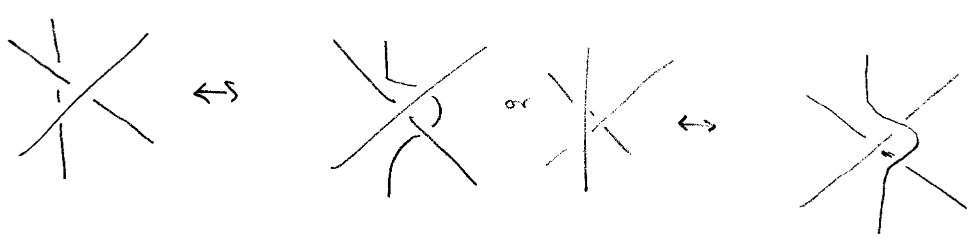
Theorem given any two projections of the same knot we can get to one from the other via planar isotopies and Reidemeister Moves.

Planar isotopy: isotopy in the plane [illegible]

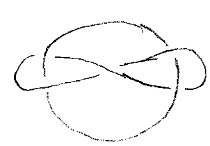
Reidemeister moves:

I:  put in a twist.

II: 

III: 

def'n a link is a set of knots tangled together

eg. unknot  whitehead 

NB. By polynomial I will mean Laurent polynomial so $\oplus$ve and a $\ominus$ve powers.

The Jones Polynomial

Development

Rule 1: $\langle O \rangle = 1$

Rule 2: $\langle \times \rangle = A \langle)(\rangle + B \langle \asymp \rangle$

$\langle \times \rangle = A \langle \asymp \rangle + B \langle)(\rangle$

flip your head 90° to see these are the same.

Rule 3 $\langle L \cup O \rangle = C \langle L \rangle$

We require these to be unchanged by R. moves.

Type II: $\langle \,)(\, \rangle = A \langle \,\rangle + B \langle \,\rangle$

$$= A \{ A \langle \,\rangle + B \langle \,\rangle \} + B (A \langle)(\rangle + B \langle$$

$$= A^2 \langle = \rangle + ACB \langle = \rangle + B^2 \langle = \rangle + AB \langle)(\rangle$$

Want

$= \langle)(\rangle$ so $B = A^{-1}$ $\quad A^2 + B^2 + C = 0$

$C = -A^2 - A^{-2}$

Type III: $\langle \,\rangle = A \langle \,\rangle + A^{-1} \langle \,\rangle$

$= A \langle \,\rangle + A^{-1} \langle \,\rangle = \langle \,\rangle$

Type II invariant.

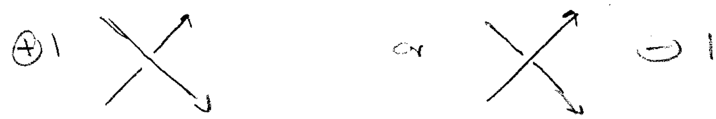
Type I: $\langle \,\rangle = A \langle \,\rangle + A^{-1} \langle \,\rangle$

$= A(-A^2 - A^{-2}) \langle \sim \rangle + A^{-1} \langle \sim \rangle$

$= -A^3 \langle - \rangle$

$\langle \,\rangle = -A^3 \langle - \rangle$ also

Give an orientation to each link.
at a crossing either



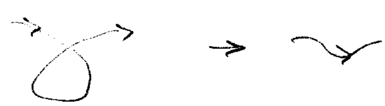
rotate the understrand clockwise to line up $\oplus 1$
" " counter clockwise " " $\ominus 1$

let $w(L) = \Sigma$ crossings. "writhe"

NB R.M I changes $w(L)$ by ± 1.

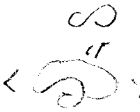
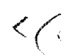
so if $X(L^*) = (-A^3)^{-w(L)} \langle L \rangle$
R.M I changes

$$X(L') = (-A^3)^{-w(L')} \langle L' \rangle$$
$$= (-A^3)^{-(w(L^*)+1)} \langle L \rangle$$
$$= (-A^3)^{-(w(L^*)+1)} (-A^3) \langle L \rangle$$
$$= (-A^3)^{-w(L)} \langle L \rangle$$

for 

So $X(L)$ is unchanged by R.M. I

since R.M. II, III don't affect $w(L)$
$X(L)$ is an invariant.
replace A by $t^{-1/4}$ gives the original Jones poly
$V(L)$.

eg. $V(O) = 1$
V(trefoil): $\langle$  $\rangle = A \langle$  $\rangle + A^{-1} \langle$  $\rangle$ 
$= A(A \langle O$  $\rangle + A^{-1} \langle$  $\rangle) + A^{-1}(A \langle$  $\rangle$
$+ A^{-1} \langle$ 

$= A^2 d(Ad + A^{-1}) + Ad + A^{-1} + Ad + A^{-1} + A^{-2}(A + A^{-1} d)$
$= A^3 d^2 + A^{-3} d + 3A^{-1} + 3Ad$
$= A^7 - A^3 - A^{-5}$

:
$+A^{-1}\langle O \rangle$
$-A^{-2}$
$\rangle$
$A^{-1}\langle (O) \rangle$
$A^{-1} d$

$w(t) =$ 

$= -3$

$$X_L = (-A^3)^{-w(L)} \langle L \rangle$$
$$= -A^9 (A^7 - A^3 - A^{-5})$$
$$= A^4 + A^{12} - A^{16}$$

$V(t)$ has $A = t^{-1/4}$

$$= t^{-1} + t^{-3} - t^{-4}$$

Right handed: $V(t) = t + t^3 - t^4$

So the trefoil is chiral. $\neq$ unknot.