Math 301 Homework 4

filename: `hmk4.problems.tex`

January 25, 2019

1. Show that $F(z) = \mathrm{Log}(-z) + i\pi$ is a branch of $\log(z)$ with branch cut on the positive real axis. Is it true that $F(z) = \mathrm{Log}_+(z)$? Here Log_+ denotes the branch of the log where the argument is chosen in $[0, 2\pi)$. (Hint: don't forget to check values right on the cut)

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2. Show that $(zw)^\alpha = z^\alpha w^\alpha$ as sets. (The set on the right is $\{a \cdot b : a \in z^\alpha, b \in w^\alpha\}$)

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3. Show that z^α

(a) is single valued if $\alpha \in \mathbb{Z}$,

(b) has q values if $\alpha = p/q$, where $p, q \in \mathbb{Z}$ with no common factors and $q > 0$. (c) has infinitely many values if α is irrational.

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4. Identify the branch points of $f(z) = \log(z(z+1)/(z-1))$. (Don't forget to check $z = \infty$.) If we define a branch for $f(z)$ by choosing the principal branch of $\log(z)$, where are the branch cuts? (Note: this example illustrates that there may be a choice of branch cuts *not* obeying our "contractible loops" condition that still result in a single valued function.)

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5. Find the branch points of $f(z) = (z^3 + z^2 - 6z)^{1/2}$. Define a branch $F(z)$ using the "range of angles" method that is continuous at $z = -1$ with $F(-1) = -\sqrt{6}$.

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6. Construct a branch $F(z)$ of $(z^2+1)^{1/2}$ that is

(i) analytic inside the unit circle,

(ii) analytic away from the imaginary axis,

(iii) equals $\sqrt{x^2+1}$ for $x \in \mathbb{R}$.

(iv) is continuous on the imaginary axis from the right.

Give an algorithm (i.e., a sequence of steps) that takes as input two real numbers x and y and computes $F(x+iy)$