1. Show that $F(z) = \log(-z) + i\pi$ is a branch of $\log(z)$ with branch cut on the positive real axis. Is it true that $F(z) = \log_+(z)$? Here $\log_+$ denotes the branch of the log where the argument is chosen in $[0, 2\pi)$. (Hint: don’t forget to check values right on the cut)

2. Show that $(zw)^\alpha = z^\alpha w^\alpha$ as sets. (The set on the right is $\{a \cdot b : a \in z^\alpha, b \in w^\alpha\}$)

3. Show that $z^\alpha$
   (a) is single valued if $\alpha \in \mathbb{Z}$,
   (b) has $q$ values if $\alpha = p/q$, where $p, q \in \mathbb{Z}$ with no common factors and $q > 0$. (c) has infinitely many values if $\alpha$ is irrational.

4. Identify the branch points of $f(z) = \log(z(z + 1)/(z - 1))$. (Don’t forget to check $z = \infty$.) If we define a branch for $f(z)$ by choosing the principal branch of $\log(z)$, where are the branch cuts? (Note: this example illustrates that there may be a choice of branch cuts not obeying our “contractible loops” condition that still result in a single valued function.)

5. Find the branch points of $f(z) = (z^3 + z^2 - 6z)^{1/2}$. Define a branch $F(z)$ using the “range of angles” method that is continuous at $z = -1$ with $F(-1) = -\sqrt{6}$.

6. Construct a branch $F(z)$ of $(z^2 + 1)^{1/2}$ that is
   (i) analytic inside the unit circle,
   (ii) analytic away from the imaginary axis,
   (iii) equals $\sqrt{x^2 + 1}$ for $x \in \mathbb{R}$.
   (iv) is continuous on the imaginary axis from the right.

Give an algorithm (i.e., a sequence of steps) that takes as input two real numbers $x$ and $y$ and computes $F(x + iy)$