

**UBC Seminar of algebraic groups, Galois
cohomology and related topics
Wednesday, January 14, 10-10:50am,
WMAX 110.**

Speaker: Baptiste Calmes (University of Cambridge)

Title: Formal group laws and flag varieties (joint work with K. Zainoulline and V. Petrov)

Abstract: The purpose of this talk is to explain how to adapt and extend the classical topological computations of the cohomology of flag varieties to the world of algebraic geometry and oriented cohomology theories (in particular algebraic cobordism).

The classical situation in algebraic topology is as follows. Let G be a compact Lie group, with maximal torus T . The flag variety G/T is a fundamental object, and it is important to understand its (say, singular) cohomology $H^*(G/T)$. To compute the ring structure of $H^*(G/T)$, one classically considers the cohomology of the classifying space $H^*(BT)$ (a power series ring) and the so-called characteristic map c from $H^*(BT)$ to $H^*(G/T)$. This ring map is surjective when the coefficients are the rational numbers, and its kernel is the ideal generated by elements fixed by the Weyl group and in positive degree. Since $H^*(G/T)$ with integer coefficients has no torsion, this is enough to give a concrete description of the ring structure on $H^*(G/T)$.

Instead, in algebraic geometry, G becomes a split semi-simple linear algebraic group with a Borel subgroup B , the flag variety is G/B and we want to replace singular cohomology by any oriented cohomology theory h . The first task will be to replace the characteristic map (since there is no algebraic BT). Then, we will explain how to compute the ring structure of $h^*(G/B)$ using the formal group law associated to h . On the way, we will encounter Schubert varieties and their desingularizations, together with Demazure and Gelfand operators.