Problem set 3. Due Friday, February 17.

Mathematics 423/502, Term 2, 2012. Instructor: Reichstein.

Throughout this assignment F will denote a field of characteristic $\neq 2$.

(1) Assume F is a finite field. Show that two non-degenerate quadratic forms over F are isomorphic if and only if they have the same dimension and the same discriminant.

(2) Chapter II, Problem 17.

(3) Prove the following partial converse to Springer's Theorem: If F is not quadratically closed (i.e., $(F^0)^2 \neq F^0$) then there exists a field extension L/F of degree 2 such that the natural homomorphism of the Witt rings, $W(F) \to W(L)$, is not injective.

(4) Let $F = \mathbb{C}(t_1, t_2)$, where $\mathbb{C}$ is the field of complex numbers and t_1, t_2 are independent variables. Show that the quadratic form $\langle 1, t_1, t_2, t_1 t_2 \rangle$ is anisotropic over F. Conclude that F is not a C_1-field. (Recall that we showed in class that F is C_2.)

(5) Chapter III. Problem 6.

(6) Chapter III. Problem 11.