Problem 1. If a generator matrix of a $q$-ary linear code $C$ has 3 rows and 15 columns, how many cosets does $C$ have?

Problem 2. Consider the ternary (i.e., 3-ary) code $C$ of length 5, consisting of all words $(a_1, a_2, a_3, a_4, a_5)$ such that
\[ a_1 + a_2 + a_3 + a_4 + a_5 = 0 \]
in $F_3$. Find a generator matrix, a parity check matrix and the minimal distance of $C$.

Problem 3. Let $C$ be a linear code.
(1) Show that the number of cosets of $C$ is $|C^\perp|$.
(2) Let $D$ be a linear code. Show that $C \subseteq D$ if and only if $C^\perp \supseteq D^\perp$

Problem 4.
Let $C$ be the linear code over $F_3$ with generator matrix:
\[ G = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \]
(a) Find a generator matrix in standard form for $C$.
(b) Find a parity check matrix for $C$.
(c) Do $(1, 0, 1, 2)$ and $(1, 1, 1, 1)$ lie in the same coset of $C$?
(d) Do $(1, 2, 0, 1)$ and $(1, 0, 2, 2)$ lie in the same coset of $C$?

Problem 5. Consider the linear code $C$ over $F_5$ defined by the parity check matrix:
\[ H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \]
(a) Find the minimal distance of $C$.
(b) Show that $C$ has 25 cosets.
(c) Show that exactly 17 of these cosets have coset leaders with weight 0 or 1.