
Problem Set 5. Due Tuesday, March 27

(1) Let \( q = 7 \) and \( C \) be a \( q \)-ary code of length 6 with parity check matrix
\[
H = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 3 & 4 & 5 & 6
\end{pmatrix}.
\]
Assuming that at most one error occurred in transmission, decode the following received words, if possible:
(a) \((1, 1, 1, 1, 1, 0)\),
(b) \((0, 0, 0, 0, 1, 1)\),
(c) \((3, 5, 0, 2, 4, 6)\).

Hint: Use a decoding scheme similar to the one in Example 7.12 in the book.

(2) Problem 7.11. Hint: Follow the solution outline in the back of the book.

(3) Find the minimum distance of the ternary linear code \( C \) with generator matrix
\[
G = \begin{pmatrix}
1 & 0 & 0 & 2 & 1 & 1 \\
0 & 1 & 0 & 1 & 2 & 1 \\
0 & 0 & 1 & 1 & 1 & 2
\end{pmatrix}.
\]

(4) Construct a generator matrix and a parity check matrix for the ternary Hamming code \( \text{Ham}(2, 3) \).

(5) Assume a codeword \( \mathbf{x} \) from for the ternary Hamming code \( \text{Ham}(2, 3) \) was sent and the word \( \mathbf{y} \) was received. Use the parity check matrix you constructed in Problem 4 to decode \( \mathbf{y} \) in each part using syndrome decoding:
(a) \( \mathbf{y} = (1, 1, 1, 0) \),
(b) \( \mathbf{y} = (2, 2, 2, 2) \),
(c) \( \mathbf{y} = (1, 2, 1, 2) \).

(6) Find the minimum distance of the binary linear code \( C \) with generator matrix
\[
G = \begin{pmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1
\end{pmatrix}.
\]

Show that \( C \neq C^\perp \) but that \( C \) is equivalent to \( C^\perp \).

(7) Recall that a linear code \( C \) is self-dual if \( C = C^\perp \). Show that the extended binary Hamming code \( \hat{\text{Ham}}(3, 2) \) is self-dual.

(8) Problem 8.10. Assume \( q \geq 3 \) is a prime.

Hint: Follow the solution outline in the back of the book.