Problem set 3. Due Thursday, March 1  

(1) Is 0131160938 a valid ISBN number?
(2) Problem 4.4.
(3) Problem 4.5.
(4) Let $C$ be the vector subspace of $F_5^4$ spanned by $(1, 1, 1, 1)$, $(1, 2, 0, 3)$ and $(4, 0, 3, 1)$. What is the dimension of $C$? Construct a generator matrix and a parity check matrix for $C$.
(5) Find a generator matrix and a parity check matrix for the ISBN code.
(6) (a) For which prime numbers $q$ is $G = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$, a generator matrix for a $q$-ary linear code $C$ in $V(6, q)$?
   (b) If $G$ is a generator matrix for $C$, determine whether or not the following words lie in $C$: $(101010)$? $(112233)$?
(7) Which of the following codes in $F_q^n$ are linear? For each linear code find a generator matrix and a parity check matrix.
   (a) $C_1 = \{(0000), (1111), (1010), (0101)\}$, $q = 2, n = 4$
   (b) $C_2 = \{(0000), (1111), (1010), (0101)\}$, $q = 3, n = 4$
   (c) $C_3 = \{000, 111, 222\}$, $q = 3, n = 3$
(8) For each of the following subsets $S$ of $GF(q)^n$, find a basis for, and the dimension of, the span of $S$. Also, determine if the given $S$ is linearly independent.
   (a) $S = \{1100, 1010, 1001, 0101\}$, $q = 2, n = 4$
   (b) $S = \{1234, 3142, 2413, 4321\}$, $q = 5, n = 4$
   (c) $S = \{0140, 4322, 1233, 2141\}$, $q = 5, n = 4$