Mathematics 342. Problem Set 4. Due Wednesday, February 15, 2012

Problem 1. Suppose $d \geq d'$. Show that if there is a q-ary (n, M, d)-code then there is a q-ary (n, M, d')-code for every $d' \leq d$, Conclude that $A_q(n, d') \geq A_q(n, d)$.

Hint: To reduce the minimal distance by (at most) one, replace the first digit by 0 in every codeword.

Problem 2. (a) Show that if there exists a q-ary (n, M, d)-code C then there exists a q-ary code $(n-1, M', d)$-code C', with $M' \geq \dfrac{1}{q}M$. In other words, $A_q(n-1, d) \geq \dfrac{1}{q}A_q(n, d)$.

(b) Use part (a) recursively to show that $A_q(n, d) \leq q^{n-d+1}$. This inequality is called the *Singleton bound*; see Theorem 10.17 in the text.

Hint: In part (a) construct C' by selecting certain codewords in C, then removing the last entry (shortening) in each of these codewords.

Problem 3. Problem 2.1 from the text.

Problem 4. Problem 2.6.

Problem 5. Let $q = 5$ and consider the q-ary linear code C of length 4, consisting of all words (a_1, a_2, a_3, a_4) such that $a_1 + 2a_2 + 3a_3 + 4a_4 \equiv 0 \pmod 5$. Find the minimal distance of C. Explain your answer.

Problem 6. Find $A_2(7, 5)$. Explain your answer.