Practice problems for midterm 2


(1) Evaluate the following expressions. Use the principal branch when the function is multivalued. Write your final answer in the form $a + bi$.

(a) $\sin(i + \frac{\pi}{2})$, (b) $(-1)^{\frac{i}{\pi}}$, (c) $\text{Log} e^{2+7i}$.

(2) Express the principal value of $(1 - i)^{4i}$ in the form $a + bi$, where $a$ and $b$ are real numbers.

(3) Find all roots of the equation $\sinh(z) = i$.

(4) Suppose the complex-valued function $f(z)$ is continuous in the entire complex plane $\mathbb{C}$ and analytic away from the origin. Show that

$$\int_{\Gamma} f(z)dz = 0,$$

for any circle $\Gamma$ centered at the origin.

(5) Let $\Gamma$ be the piece of the parabola $y = x^2$ from $0$ to $2 + 4i$. Find

$$\int_{\Gamma} |z|^2dz.$$

(6) Show that the function $f(z) = \overline{z}$ does not have an anti-derivative in the complex plane.

(7) Evaluate

$$\int_{\Gamma} \frac{\sin(z)}{z^2(z-3)} dz,$$

where $\Gamma$ is the positively oriented unit circle, $|z| = 1$.

(8) Evaluate

$$\int_{\Gamma} \frac{1}{(z-2)^2} dz.$$ 

Here $\Gamma$ is the upper half of the unit circle, $|z| = 1$, $\text{Im}(z) \geq 0$, with initial point $-1$ and terminal point $1$.

(9) For each function $f(z)$ below evaluate

$$\int_{\Gamma} f(z)dz,$$

where $\Gamma$ is the positively oriented circle of radius $2$ centered at the origin.

(a) $f(z) = \frac{\sin(z)}{2z}$, (b) $f(z) = e^z \cos(z)$, (c) $f(z) = \overline{z}$, (d) $f(z) = \frac{e^{iz}}{(z+i)^3}$.

(10) Suppose $f(z)$ is an entire function such that $|f(z)| < 2|z|^2 + 5$ for all complex numbers $z$. Show that $f(z)$ is a polynomial of degree $\leq 2$.

Hint: Use the Cauchy estimate for $|f'''(z)|$ to show that the third derivative $f'''(z)$ is identically zero.