Mathematics 300, Section 201, Practice problems for Midterm 1

**Problem 1:** Find the modulus and argument for each of the complex numbers below. Give the unique value of the argument that lies in the interval $[0, 2\pi)$.

(a) $\frac{2}{i} + \frac{i}{5}$.
(b) $\left(\frac{1 + i\sqrt{5}}{2}\right)^{3000}$.

**Problem 2:** Find all solutions of the equation $z^4 = 8iz$ and express them in the form $a + ib$ where $a$ and $b$ are real numbers.

**Problem 3:** Let $v(x, y) = 5x - xy + 4$.
(a) Show that $v(x, y)$ is harmonic in the entire plane.
(b) Construct an entire function $f(z)$ such that $\text{Im}(f(z)) = v(x, y)$.

**Problem 4:** (a) Show that if $f(z)$ and $\overline{f(z)}$ are both analytic in a domain $D$, then $f(z)$ is constant in $D$.
(b) Using part (a), show that $p(z)$ is not analytic in any domain of the complex plane if $p$ is a polynomial with degree at least 1.

**Problem 5:** Find the partial fraction decomposition of $R(z) = \frac{2}{z(1 - z)^2}$.

**Problem 6:** For each of the statements below, indicate whether they are true or false. If true, give a proof. If false, give a counter example.

(a) $|e^{-z}| \leq 1$ if $|z| \leq 1$.
(b) $\text{Arg}(\text{Re}(z)) = 0$ for any complex number $z$. Here $\text{Arg}$ denotes the value of the argument that lies in the interval $(-\pi, \pi]$.
(c) The equation $e^z = -1$ has no complex solution.
(d) $f(z) = \frac{\bar{z} - 1}{|z|^2 - z}$ is a rational function.
(e) $-z^4 - 1 < 0$ for every complex number $z$. 