Recall that
\[ \sec(x) = \frac{1}{\cos(x)}, \quad \csc(x) = \frac{1}{\sin(x)}, \quad \text{and} \quad \cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}. \]

1. Find the derivatives of the following trigonometric functions.
   (a) \( f(x) = \tan(x) \)

   **Solution:** Since \( \tan(x) = \frac{\sin(x)}{\cos(x)} \), from the Quotient rule we have that
   \[
   \frac{d}{dx} \tan(x) = \frac{d}{dx} \left( \frac{\sin(x)}{\cos(x)} \right) = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x).
   \]

   (b) \( f(x) = \sec(x) \)

   **Answer:** \( \frac{d}{dx} \sec(x) = \sec(x) \tan(x) \).

   (c) \( f(x) = \csc(x) \)

   **Answer:** \( \frac{d}{dx} \csc(x) = -\csc(x) \cot(x) \).

   (d) \( f(x) = \cot(x) \)

   **Answer:** \( \frac{d}{dx} \cot(x) = -\csc^2(x) \).

2. Differentiate the following functions.
   (a) \( f(x) = \sin^3(x) + \tan^3(x) \)

   **Solution:** From the Chain rule and the derivative calculations from (1) we have that
   \[
   f'(x) = 3\sin^2(x) \cdot \cos(x) + 3\tan^2(x) \cdot \sec^2(x) = 3\sin^2(x) \cos(x) + 3\tan^2(x) \sec^2(x).
   \]

   (b) \( f(x) = \left( \frac{\cos(x)}{x^2 \sin(x)} \right)^4 \)

   **Answer:**
   \[
   f'(x) = 4 \left( \frac{\cos(x)}{x^2 \sin(x)} \right)^3 \cdot \frac{-\sin(x) \cdot x^2 \sin(x) - \cos(x) (2x \sin(x) + x^2 \cos(x))}{x^4 \sin^2(x)}.
   \]
(c) \( f(x) = x^{\cos(x)} \)

Solution: Taking the natural logarithm of both sides we have \( \log(f(x)) = \cos(x) \log(x) \). Hence, implicit differentiation yields

\[
\frac{1}{f(x)} f'(x) = -\sin(x) \log(x) + \cos(x) \cdot \frac{1}{x}.
\]

Therefore,

\[
f'(x) = x^{\cos(x)} \left( \frac{\cos(x)}{x} - \sin(x) \log(x) \right).
\]

3. Find the equation of the line tangent to \( y = \log(1 + \cos^2(x)) \) at \( x = \frac{\pi}{3} \).

Answer: \( y = \log \left( \frac{5}{4} \right) - \frac{2}{\sqrt{3}} \left( x - \frac{\pi}{3} \right) \).

4. A runner sprints around a circular track of radius 100m at a constant speed of 7m/s. The runner’s friend is standing at a distance 200m from the centre of the track. How fast is the distance between the friends changing when the distance between them is 200m?

Answer: \( \frac{7\sqrt{15}}{4} \) m/s. **Hint:** Use the Law of Cosines.