MATH 105 – 951
MATH 105 Midterm Practice Problems

Short Answer Questions
Evaluate the following integrals or state that they diverge.

1. \( \int_0^5 \frac{x}{x + 10} \, dx \)

2. \( \int_3^4 \frac{1}{y^2 - 4y - 12} \, dy \)

3. \( \int_0^a \frac{1}{\sqrt{a^2 - x^2}} \, dx \)

4. \( \int_2^\infty \frac{1}{x \ln(x)} \, dx \)

5. \( \int \tan^2(u) \cos^2(u) \, du \)

6. \( \int \csc^4(\theta) \cos(\theta) \, d\theta \)

7. \( \int \frac{x^2 + 2}{x + 2} \, dx \)

8. \( \int t \cos(t^2) \, dt \)

9. \( \int x^{3/2} \ln(x) \, dx \)

10. \( \int \ln(x) \, dx \) (Hint: Think of \( \ln(x) \) as \( 1 \cdot \ln(x) \).)

11. \( \int_1^2 \frac{1}{\sqrt{x - 1}} \, dx \)
Long Answer Questions

1. Let \( F(x) = \int_{1}^{x} \frac{1}{t^2 + 6t + 5} dt \). Find the equation of the line tangent to \( F(x) \) at \( x = 2 \).

2. Recall that a differentiable function \( F(x) \) is said to be increasing at a point \( a \) if \( F'(a) \geq 0 \). Show that the function 
\[
\int_{0}^{e^x} e^t dt
\]
is always increasing.

3. Find the derivative of the function \( F(x) = \int_{0}^{x} xe^t dt \). (Hint: \( x \) does not depend on \( t \).)

4. If \( f(x) \) is continuous on \([a, b]\), then the average value of \( f(x) \) on \([a, b]\) is defined to be 
\[
f_{\text{ave}} = \frac{1}{b-a} \int_{a}^{b} f(x)dx.
\]
Find the average value of the function 
\[
f(x) = \frac{x^3 + 4}{x^2 + 4x + 3}
\]on the interval \([0, 2]\).

5. Find the area of the overlapping portion of the circles \( x^2 + (y-1)^2 = 1 \) and \( x^2 + y^2 = 1 \) that is in the first quadrant. (Note: The equation \( x^2 + (y-1)^2 = 1 \) is the equation of the circle of radius 1 centred at the point \((0, 1)\), and the equation \( x^2 + y^2 = 1 \) is the equation of the circle of radius 1 centred at the origin.)

6. Find the area between the curves \( y = e + \sin^2(\pi x) \) and \( y = xe^x \) in the first quadrant.
7. (a) Use sigma notation to write down a Midpoint Riemann Sum approximation of the area under the curve \( y = e^{1/t} \) between \( t = 1 \) and \( t = 2 \) with \( n = 10 \). Do not evaluate the Riemann sum.

(b) Use the error formula

\[
|\text{error}| \leq \frac{N(b-a)^3}{24n^2}
\]

to find a bound for the error in the approximation in (a).

8. The lifetime \( T \) (in hours) of a lightbulb has probability density function

\[
f(t) = \begin{cases} 
\frac{1}{100}e^{-t/100} & \text{if } t \geq 0 \\
0 & \text{if } t < 0 
\end{cases}
\]

(a) Find the probability that the lightbulb will not burn out within the first 100 hours.

(b) Find the expected lifetime of the lightbulb.

9. The distance \( X \) (in cm) between a dart’s location on a dartboard and the bullseye (centre of the dartboard) has probability density function

\[
f(x) = \begin{cases} 
\frac{3}{4000}(20x - x^2) & \text{if } 0 \leq x \leq 20 \\
0 & \text{otherwise} 
\end{cases}
\]

Calculate the standard deviation in \( X \), \( \sigma(X) \).