1. State whether the following are true or false. If true, provide a short justification. If false, provide a counterexample.

(a) If the sequence \( \{(-1)^n a_n\} \) converges, then the sequence \( \{a_n\} \) converges to 0.

(b) If the sequence \( \{a_n\} \) diverges, then \( \lim_{n \to \infty} a_n = \pm \infty \).

(c) If the series \( \sum_{n=1}^{\infty} a_n \) converges, then \( \sum_{n=1}^{\infty} \cos(a_n) \) converges.

2. The sequence \( \{a_n\} \), where \( a_n = (1 + \frac{x}{n})^n \), converges for all real numbers \( x \) to some number \( a_x > 0 \). Find \( a_x \) (your answer will be in terms of \( x \)).

3. The set \( F \) is a fractal constructed as follows.
   
   (i) Begin with the line segment \( I_0 = [0, 1] \).
   (ii) Remove the open middle third \( \left(\frac{1}{3}, \frac{2}{3}\right) \), to get \( I_1 = [0, \frac{1}{3}] \cup \left[\frac{2}{3}, 1\right] \).
   (iii) Remove the open middle third from every remaining line segment, to get
   \[
   I_2 = \left[0, \frac{1}{9}\right] \cup \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \frac{7}{9}\right] \cup \left[\frac{8}{9}, 1\right].
   \]
   (iv) Repeat the process ad infinitum (forever).
   A few iterations of this construction are illustrated below.
(a) By considering a suitable series, show that the total length of all of the intervals removed is equal to 1.

(b) If the total length removed is 1, have we removed all of the points from the interval [0, 1]? Justify your answer.

4. (Bonus) State whether the following series is convergent or divergent. If it is convergent, find its sum.

\[
\sum_{n=1}^{\infty} \frac{1}{n^3 + 3n^2 + 2n}
\]