For this assignment, you are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence of your solutions. You are encouraged to work together, but your solutions must be written independently. Please write your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

This assignment is due at 1:00pm on Wednesday, July 13. Late assignments will not be accepted.

1. Evaluate the limit

$$\lim_{{h \to 0}} \frac{\int_{2}^{2+h} e^{-t^2} \, dt}{h}.$$  

Hint: First rewrite the integral as

$$\int_{2}^{2+h} e^{-t^2} \, dt = \int_{2}^{1} e^{-t^2} \, dt + \int_{1}^{2+h} e^{-t^2} \, dt$$

$$= \int_{1}^{2+h} e^{-t^2} \, dt - \int_{1}^{2} e^{-t^2} \, dt$$

Next, let \( F(x) = \int_{1}^{x} e^{-t^2} \, dt \), and write this difference of integrals (above) in terms of \( F(x) \). Now, recognize the limit as something from MATH 104.

2. Find an antiderivative of the function \( f(t) = \sin^3(t) \cos^5(t) \).

3. Use a method of integration to show that a circle of radius \( r \) centred at the origin has area \( \pi r^2 \). (Hint: Because of the symmetry of circles, it is sufficient to show that the area of the quarter circle in the first quadrant is \( \frac{\pi r^2}{4} \).)

4. Suppose \( f(x) \) is such that \( f''(x) \) is continuous on \([0, \pi]\), \( f(\pi) = 1 \) and

$$\int_{0}^{\pi} (f(x) + f''(x)) \sin(x) \, dx = 2.$$  

Find \( f(0) \).

Hint: Break up the integral into two pieces:
\( \int_{0}^{\pi} (f(x) + f''(x)) \sin(x) \, dx = \int_{0}^{\pi} f(x) \sin(x) \, dx + \int_{0}^{\pi} f''(x) \sin(x) \, dx \).

Use a method of integration to find a relationship between \( \int_{0}^{\pi} f(x) \sin(x) \, dx \) and \( \int_{0}^{\pi} f''(x) \sin(x) \, dx \).