1) Find the greatest common divisor of 20785 and 44350.

2) Show that the integers 34709 and 100313 are relatively prime and use the Euclidean algorithm to express 1 as an integral linear combination if these two integers in two different ways.

3) Show that for any integer \( k \geq 0 \), the integers \( 3k + 2 \) and \( 5k + 3 \) are relatively prime.

4) Two players begin with a pair of positive integers and take turns making moves of the following type: A player can move from the pair of positive integers \((x, y)\) with \( x \geq y \), to any of the pairs \((x - ty, y)\), where \( t \) is a positive integer and \( x - ty \geq 0 \). A winning move consists of moving to a pair where one element equals zero. Show that every sequence of moves starting with the pair \((a, b)\) must eventually end with the pair \((0, (a, b))\).

5) Let \( \alpha = a + b\sqrt{-5} \) where \( a \) and \( b \) are integers. Define the Norm of \( \alpha \), denoted \( N(\alpha) \) as \( N(\alpha) = a^2 + 5b^2 \). Show that if \( \alpha = a + b\sqrt{-5} \) and \( \beta = c + d\sqrt{-5} \), with \( a, b, c, d \in \mathbb{Z} \), then \( N(\alpha\beta) = N(\alpha)N(\beta) \).

6) Let \( R = (a + b\sqrt{-5}) \) where \( a, b \) are integers. Use Exercise 5) above to show that 2 is a prime number in \( R \) (i.e. if 2 can be written as a product \( 2 = u \cdot v \), with \( u, v \in R \), then either \( u \) or \( v \) should be equal to \( \pm 1 \)). Show also that there are two distinct prime factorisations of 21 in \( R \).

7) Find the LCM and GCD of 343 and 999. Show that if \( a \) and \( b \) are positive integers such that \((a, b) = 1\), then \((a^n, b^n) = 1\) for any positive integer \( n \).

8) Use the Fundamental Theorem of Arithmetic to show that \( \sqrt{2} + \sqrt{3} \) is not a rational number.

9) Show that \( \log_2 3 \) is an irrational number.

10) For \( a, b \) positive integers, suppose that the gcd \((a, b)\) and lcm \([a, b]\) are equal. Then show that \( a = b \).