1) \(1! + 2! + 3! + \cdots + 100! \equiv 11 + 2! + \cdots + 6! \mod 7\), since \(7 \mid k!\) for \(k \geq 7\). The LHS is congruent to \(1 + 2! + 3! + \cdots + (-1)\) modulo 7, which is congruent to \(2 + 6 + 3 + 1\) modulo 7. So the LHS is congruent to 5 modulo 7.

2) A Carmichael number is a composite integer \(n\) that satisfies \(b^{n-1} \equiv 1 \mod n\), for all positive integers \(n\) such that \((b,n) = 1\).

OR

A composite integer \(n\) that is a pseudoprime to every base \(b\) with \((b,n) = 1\) is a Carmichael number.

\[2821 = 7 \times 13 \times 21\]

and \((7-1) = 6 \mid 2820 = 2820 - 1, 12 \mid 2820, 20 \mid 2820\). Hence 2821 is a Carmichael number.

3) 
\[17^4 \equiv 1 \mod 45.\]

Writing \(45 = 44+1 = 4.11+1\), we see that \(17^{44} \equiv 1 \mod 45\) and hence 45 is a pseudoprime to base 15. Also \(17^{45} \) is congruent to 17 modulo 45.

4) Observe that \(3^4 = 81\) which is congruent to 1 modulo 10. Hence \(3^{1000}\) is congruent to 1 modulo 10, so the last digit in the decimal expansion is 1.

5) 
a) TRUE; \(2^5 - 1 = 31\) and 5 divides 60.
b) TRUE: Theorem 3.8, since \((3,8)=1\).
c) TRUE: \(\phi(1)=1\) is odd.
d) TRUE: For \(n = 200\), \(\tau(n) = 12\).