1) Show that if $n$ is a positive integer, then $\phi(2n) = \phi(n)$ if $n$ is odd and $\phi(2n) = 2\phi(n)$ if $n$ is even. (5 pts)

2) Show that if $k > 0$, then $\phi(n) = k$ has only finitely many solutions. (4 pts)

3) Let $f$ be the arithmetic function defined by $f(n) = \phi(n)/n$. Show that $f(p^k) = f(p)$ for all primes $p$ and all positive integers $k$. (3 pts)

4) Use Euler’s theorem to evaluate $2^{10000} \mod 77$. (4 pts)

5) Prove that if $n$ is a positive integer, then $\phi(3n) = 3\phi(n)$ if and only if $3 \mid n$. (4 pts)